Inventory, Speculators, and Initial Coin Offerings*

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Abstract

Initial Coin Offerings (ICOs) are an emerging form of fundraising for Blockchain-based startups. We propose a simple model of matching supply with demand with ICOs by companies involved in production of physical products. We examine how ICOs should be designed—including optimal token floating and pricing for both the utility tokens and the equity tokens (aka, security token offerings, STOs)—in the presence of product risk and demand uncertainty, make predictions on ICO failure, and discuss the implications on firm operational decisions and profits. We show that in the current unregulated environment, ICOs lead to risk-shifting incentives (moral hazard), and hence to underproduction, agency costs, and loss of firm value. These inefficiencies, however, fade as product margin increases and market conditions improve, and are less severe under equity (rather than utility) token issuance. Importantly, the advantage of equity tokens stems from their inherent ability to better align incentives, and hence continues to hold even in unregulated environments.

Keywords: Initial Coin Offering (ICO), Security Token Offering (STO), Inventory, Strategic Agents, Moral Hazard, Crowdfunding, Firm Operations

1 Introduction

Initial Coin Offerings (ICOs) are an emerging form of fundraising for blockchain-based startups in which either utility and/or equity tokens are issued to investors in exchange for funds to help finance business[1]. This new way of crowdfunding[2] startup projects has gained a lot of momentum since 2017 with the total amount raised skyrocketing to form a ten-billion-dollar market as of November 2018 (Coinschedule 2018). The growth of ICOs is also challenging the dominance of traditional means of raising capital. During Q2 2018, ICO projects raised a total volume of $9.0 billion (Coinschedule 2018), which is 56% of the amount raised by the US IPO market ($16.0 billion) or 39% of the amount raised by the US venture capital markets ($23 billion) during the same period, as reported by CB Insights (CB Insights 2018) and PwC (Thomson 2018).

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[1] In many cases, tokens are created on existing blockchains and their core value is backed by the firm's products or services.

[2] We discuss differences between token offerings and other early-stage financing methods in Appendix A.2.

Following this trend, the academic literature on ICOs is also rapidly growing, particularly in finance and economics, where the focus has been on topics such as empirically characterizing the drivers of ICO success or on comparing this new form of financing to more traditional financing methods (see literature review). There is also a growing literature in operations management studying the interplay between firm operations and financing decisions, though the focus here has been mostly on more traditional financing methods, such as debt and equity (Xu and Birge 2004). This paper seeks to contribute to these literatures by focusing on the largely unexplored implications of ICOs for product market firms facing demand uncertainty, in unregulated markets. In particular, we ask: How should ICOs be designed as a function of product, firm and demand characteristics? That is, what type of tokens, and how many of them should be issued, and how should they be priced? Further, how do these choices affect firm inventory decisions, and the odds of ICO failure or success? Finally, what are some of the salient features distinguishing ICOs from other forms of financing?

A typical ICO proceeds as follows. A startup first publishes a white paper with or without a minimum viable product for demonstration and then issues its platform-specific tokens. The typical white paper usually delivers the key information of the project, including the token sale model that specifies the token price, the sale period, the sales cap (if any), etc.[3] The tokens can have a variety of uses, but most commonly, they are either used for consumption of the company's goods and services (utility tokens), or offered as shares of the company's future profit (equity tokens). During the crowdsale, investors purchase tokens using either fiat currencies, or, more commonly, digital currencies such as Bitcoin and Ether.

While some successful ICOs were conducted by service platforms such as Ethereum and NEO, in this paper we focus on ICO projects that involve the delivery of physical products instead; these types of ICOs are just now starting to emerge, and hence, are less well-understood. One striking example is that of Sirin Labs (Sirin Labs 2019): a startup that produces smartphones and other types of hardware systems. In 2017, Sirin Labs was able to raise over $150 million from investors by offering them Sirin tokens (SRN). These tokens could subsequently be used to purchase the company's products and participate in its ecosystem, or be sold in the secondary market. Perhaps one of the reasons that the ICO raised such a staggering amount was that the startup had secured Foxconn as a major supplier, and hence, risk of product failure was arguably low. But while Sirin Labs indeed went on to successfully manufacture its product, its market cap fell drastically in the months and years post its ICO, as demand for the phones fell well short of expectations. Other

[3] We provide a condensed example of a white paper in Appendix A.3.

relevant examples include Honeypod (Honeypod 2018) that develops hardware serving as the main hub interconnecting various devices and providing traffic filtering, and Bananacoin (Bananacoin 2018) which grows bananas in Laos.

ICOs can have multiple benefits. First, both the startup founders and investors have the opportunity of high financial gains from the potential appreciation of the tokens. Second, ICOs allow for faster and easier execution of business ideas because the ICO tokens generally have secondary-market liquidity, and require far less paperwork and bureaucratic processes, than the regulated capital-raising processes do. Third, ICOs provide the project team with access to a larger investor base as the participants (typically) face less location restrictions.

On the other hand, just like the underlying blockchain technology, ICOs are still in their infancy and have some downsides. First, for the project teams, the failure rate of ICOs is high and increasing. Despite a rise in the total investment volume, nearly half of all ICOs in 2017 and 2018 failed to raise any money at all (Seth 2018) and 76% of ICOs ending before September 2018 did not get past their soft cap (Pozzi 2018), i.e., the minimum amount of funds that a project aims to raise. Benedetti and Kostovetsky (2018) claim that only 44.2% of the projects remain active on social media into the fifth month after the ICO. Second, the aspect of quick and easy access to funding with loose regulation attracts unvetted projects and even utter scams, making ICO investments risky. Some entrepreneurs portray deceiving platform prospects in the white papers in an attempt to raise as much money as they can before gradually abandoning their projects. In a review of 1450 ICO cases by the Wall Street Journal, 271 were susceptible to plagiarism or fraud. The profit-seeking yet ill-informed investors can become easy prey and have claimed losses of up to $273 million (Shifflett and Jones 2018). Other disadvantages of ICOs include technical concerns such as the potential theft of tokens through hacks (Memoria 2018).

To study some of these issues we use a game-theoretic approach and construct a three-period ICO model with stochastic demand for a product, adapting the classical newsvendor inventory model (Arrow et al. 1951). There are three types of players in the game: a firm (token issuer), speculators (token traders) and customers (who buy the product). As in the Sirin Labs example mentioned earlier, the firm seeks to raise funds through an ICO to support the launch of a physical product it wishes to sell to customers, and the main source of risk is that of future demand uncertainty. In the first period, the firm announces the total number of tokens available, the sales cap and the ICO token price, and sells tokens (up to the cap) to speculators who make purchase decisions strategically[4]. In the second period, the firm, facing uncertain customer demand, can put

[4] The literature on operational decisions in the presence of strategic agents includes Dana Jr and Petruzzi (2001),

the funds raised in the ICO towards production of a single product. Importantly, to reflect the lack of intermediaries and the lax regulatory environment, we leave the firm with full discretion over what to do with the raised funds, including the option of fully shirking production and diverting raised funds to its pockets (moral hazard). In the final period, demand for the product is realized and customers buy tokens either directly from the firm (if the firm has any tokens remaining) or from the speculators in the secondary market, and redeem these tokens for the product, if available, at the equilibrium token price. To incorporate additional salient features, we extend the model by considering risks of production failure and speculator outside investment options in §5.

Using this relatively simple and flexible model, we derive the optimal ICO price, ICO token cap and production quantity as a function of operational and demand characteristics. We examine two types of tokens—the utility tokens and the equity tokens (more commonly known as security token offerings—STOs).[5] We find that despite rampant moral hazard, both product-based utility and equity ICOs can be successful under the right conditions. The existence of a secondary market is crucial to this end, as it provides incentives for speculators to participate in the ICO, even when they might otherwise not be interested in consuming the firm's product. Of course, moral hazard understandably leads to agency costs, and hence underproduction and lower-than-optimal profits versus first best, especially when the demand distribution of the product has large variance. For ICOs to be able to overcome these inefficiencies, we show that they require high price-cost ratios of the product and a minimum fraction of tokens sold during the first round of sale. Interestingly, excessive funds raised from over-optimistic investors (e.g., when more than half of all tokens are sold during the first round) may directly discourage production after funds are raised. Our comparative-statics analysis suggests that these inefficiencies can be further reduced when the demand for the product is higher, less volatile or (and) when the customers' willingness-to-pay is higher. Moreover, we find such inefficiencies to be less prominent for ICO's with equity tokens (STOs) as these almost achieve the first-best outcome under favorable market conditions.

Our model distinguishes ICOs from other early-stage financing methods by capturing several unique features of ICOs, including the fundraising mechanism and the issuance of tokens, the existence of a peer-to-peer secondary market, and the nature of the investors. In contrast to reward-based crowdfunding, for instance, there is no intermediary platform imposing an all-or-nothing mechanism. Rather, firms running ICOs have to determine how many tokens to issue/sell during the first round in addition to how many products to make. In the case of utility tokens, we

Cachon and Swinney (2009), Papanastasiou and Savva (2016), etc. Su (2010) and Milner and Kouvelis (2007) consider similar speculative behavior yet with no financing aspect.

[5] Throughout the paper, we use "STOs" and "ICOs with equity tokens" interchangeably.

show this is akin to deciding how much of the firm's future revenues to share with speculators. In the case of equity tokens, this is akin to deciding how much of the firm's future profits to share. Another important difference that we highlight is that tokens allow the firm to disperse downside risks of future demand among the token holders, whereas in crowdfunding, campaign backers share downside risk only in terms of product failure (not in terms of future demand uncertainty). Finally, we show that the existence of the secondary market for the tokens is crucial in incentivizing investors to participate in ICOs, an important feature missing from crowdfunding. We refer the readers to Appendix A.2 for a more detailed discussion and Table 1 for a summary comparison to other financing methods.

Literature Review Broadly speaking, this paper contributes to the strand of literature at the interface of operations and finance that studies, among other things, different ways of financing inventory. Earlier works include Babich and Sobel (2004), Buzacott and Zhang (2004), Boyabatlı and Toktay (2011), Kouvelis and Zhao (2012), and Yang and Birge (2013), see Kouvelis et al. (2011) for a review of this literature. More recent papers include Boyabatlı et al. (2015), Yang et al. (2016), Iancu et al. (2016), Alan and Gaur (2018), Chod et al. (2019).

As an alternative to traditional crowdfunding,[6] ICOs are understudied in the operations management literature. However, there are several recent theoretical studies in the finance literature that examine the economics of ICOs and cryptocurrencies. Most of them focus on peer-to-peer service platforms that allow decentralized trading. For example, Li and Mann (2018) and Bakos and Halaburda (2018) demonstrate that ICOs can serve as a coordination device among platform users. In a dynamic setting, Cong et al. (2018) consider token pricing and user adoption with inter-temporal feedback effects.

More closely related to our work are papers that model ICOs in business-to-customer settings. Catalini and Gans (2018) propose analysis of an ICO mechanism whereby the token value is derived from buyer competition. Chod and Lyandres (2018) study the extent to which risk-averse entrepreneurs can transfer venture risk to fully diversified investors under ICO financing. These papers also analyze the distinctions between the economics of ICO financing and those of traditional equity financing. We adopt a similar approach modeling an ICO as a presale of the platform's partial future revenue, yet with an emphasis on operational details including demand uncertainty and inventory considerations. In particular, we incorporate stochastic demand of the products, rather than assuming that demand is observable before production (Catalini and Gans 2018) or infinite

[6]Refer to Section A.2 for a discussion on the differences. For recent papers on crowdfunding, see Alaei et al. (2016), Babich et al. (2019), Belavina et al. (2019), Chakraborty and Swinney (2017), Chakraborty and Swinney (2018), Fatehi et al. (2017), Xu and Zhang (2018), Xu et al. (2018).

(Chod and Lyandres 2018). We believe ours is the first study to jointly optimize the operational decisions including sales cap, token pricing and production quantity, in the presence of strategic investors under demand uncertainty, and compare utility and equity (STO) token issuance in this context.

Here and below, we first develop in §2 and solve in §3 the case of utility tokens, before examining equity tokens as an extension in §4.

2 Model: Utility Tokens

Consider an economy with three types of agents: i) a monopolist firm, ii) investors termed speculators, and iii) firm's customers. The economy has three periods: i) The first period, termed "ICO", is the fundraising phase containing the firm's white paper that includes contract terms and the token crowdsale; ii) the second period, termed "production", covers firm's production decisions in the face of uncertain customer demand; iii) the third period, termed "market", covers the realization of customer demand, and market clearing for the product and any remaining tokens. The firm participates in all three periods. Speculators participate in the ICO and the market periods. Customers participate only in the market period.

Firm The firm has no initial wealth and seeks to finance production through a "capped" ICO. The firm has a finite supply of m total tokens that are redeemable against its future output (if any). In the ICO period, the firm maximizes its profits by choosing i) the ICO "cap", $n \leq m$, that is, the number of tokens to sell to speculators in the ICO period, and ii) the ICO token price τ (in dollars per token). Subsequently, in the production period, the firm has the option to use any amount of funds raised through the ICO to finance the production of its output. To this end, the firm maximizes its total wealth, through a newsvendor-type production function (Arrow et al. 1951), by choosing quantity $Q \geq 0$ of a product with unit cost c (in dollars per unit) that it can later sell in the market period at a price p (in tokens per unit), in the face of uncertain customer demand D. To capture the lack of regulation in the current environment, we assume that the firm could divert all or a portion of the funds raised through the ICO, rather than engage in production (moral hazard).

In the final market period, demand is realized and the product is launched. The product can only be purchased using the firm's tokens—a restriction that has two consequences: i) it endows tokens with (potential) value ii) it implies price p represents the exchange rate between tokens and units (which departs from the traditional newsvendor (Arrow et al. 1951) setting). The firm competes with speculators to sell any remaining tokens it has post-ICO to product customers, e.g., through a "secondary" offering round. As opposed to the ICO round, there is no uncertainty in the secondary offering round as production is finished and demand is already realized. The equilibrium token price τ_{eq} (in dollars per token) as well as the product price p (in tokens per unit) are then derived through a market clearing condition, described below. Once the market clears, tokens have no residual value (since there is only a single production round and the tokens have no use on any other platform) and the game ends. We provide more details of the tokens' features and discuss their implications for speculators and customers in Appendix A.1.

To recap, the firm's decisions are the number of tokens to make available in the ICO to speculators n, the ICO token price τ, and production quantity Q.

Speculators Let z denote the total number of speculators with $z \gg m$ reflecting that ICOs have low barriers to entry. Speculators are risk-neutral, arrive simultaneously, and can each try to purchase a single token in the ICO at the price set by the firm, τ, that they expect to subsequently sell in the market period at an equilibrium price $\mathbb{E}[\tau_{eq}]$, where $\mathbb{E}$ is the expectation operator. If demand for tokens exceeds token supply in the ICO, speculators are randomly allocated token purchase rights. Speculators' expected profit u depends, among other things, on the expected price difference $\mathbb{E}[\tau_{eq}] - \tau$, denoted Δ, and on the total number of speculators that purchase tokens in the ICO, denoted s; formally:

$$u(s) = \frac{s}{z}\Delta(s), \quad \text{with} \quad \Delta(s) = \mathbb{E}[\tau_{eq}(s)] - \tau, \tag{1}$$

where the ratio s/z reflects random assignment of token purchase rights. We emphasize that the number of speculators s will be determined endogenously in equilibrium, and as we shall show later on, this number depends on the ICO cap n and the ICO token price τ. A necessary condition for $s(\tau, n) > 0$ speculators to participate in the ICO is

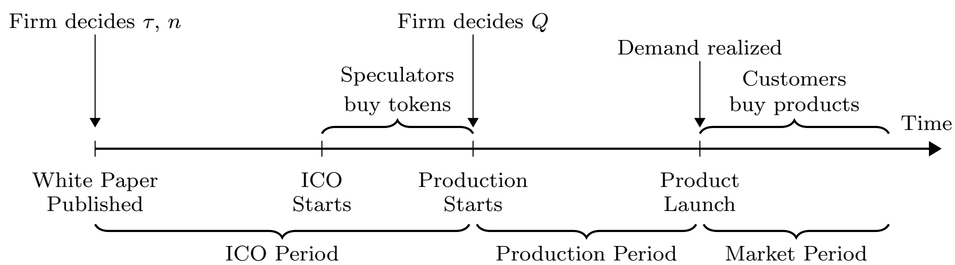
$$u(s(\tau, n)) \geq 0 \quad \text{(participation constraint).} \tag{2}$$

Note, as we show in Appendix B.3, assuming sequential rather that simultaneous arrival of speculators does not impact the results of the paper. The proofs are written to cover both cases. Also note that the model readily extends to the case in which speculators are given the additional option of using their tokens to purchase the firm's product rather than selling their tokens to product customers.

Product Customers Customers who join the market after the product launch have a homogeneous willingness-to-pay v (dollars per unit) for the product that is strictly greater than the production cost c. As we shall see later on, v plays a critical role in the market clearing condition. Customers can buy tokens directly from the firm (if it has any tokens remaining in the market period) or from speculators, and they can redeem the tokens for the products. The demand for the product D is stochastic and we denote the cumulative distribution function of demand by $F(\cdot)$. For ease of analysis, we assume that $F(\cdot)$ is continuous and $F^{-1}(0) = 0$.

We summarize the timeline in Figure 1 below.

Figure 1: Sequence of Events



Market clearing Clearing occurs in the market period. Recall that the customers have a constant willingness-to-pay v (dollars per unit). This means that the dollar-denominated price of the product charged by the firm, which is equal to the product of the token-denominated price of the product p (tokens per unit) and the equilibrium market token price τ_{eq} (dollars per token), is at most v. Since the firm is a monopolist, it sets the dollar-denominated price to be exactly v, i.e., $p \cdot \tau_{eq} = v$. Therefore, p and τ_{eq} have an inverse relationship, and we have the following lemma due to the law of supply and demand.

Lemma 1. (Equilibrium Prices)

i) The equilibrium token-denominated price of the product is $p = m/\min\{Q, D\}$.

ii) The equilibrium token price in the market period is given by $\tau_{eq} = \frac{v}{m}\min\{Q, D\}$.

Part (i) of Lemma 1 suggests that there is no idle token in the market period—all m tokens are redeemed for $\min\{Q, D\}$ products. Therefore, customers' valuation of all tokens is equivalent to their willingness-to-pay for all products that are purchased by these tokens, i.e., $\tau_{eq}\, m = v \min\{Q, D\}$. Part (ii) addresses one of the most frequently asked questions regarding ICOs—what gives tokens their ultimate value. In our model, the value of platform-specific tokens depends positively on three factors: the quality of the product reflected by the customers' willingness-to-pay, the sales volume determined by the supply and demand for the products and the scarcity of tokens inversely determined by the total supply, m.

Note, the term $v\,\mathbb{E}[\min\{Q,D\}]$ resembles the revenue term in the traditional newsvendor setup (Arrow et al. 1951) where v corresponds to the fixed price. While a traditional newsvendor sells a quantity of products at a fixed price, the firm in our model sells a fixed number of tokens at (or below, to satisfy the participation constraint) a market equilibrium token price. However, the newsvendor form emerges from the fact that τ_{eq} is tied to the product sales volume $\min\{Q,D\}$ via the market clearing condition.[7]

Firm's optimization problem The firm maximizes its expected dollar-denominated wealth at the end of the market period, denoted by Π, which consists of three terms: i) the total funds raised during the ICO, $\tau s(\tau,n)$, plus ii) the expected total funds raised in the secondary offering, $(m-s)\mathbb{E}[\tau_{eq}]$, minus iii) production costs $c\,Q$. The constraints are i) that production is funded by funds raised in the ICO, i.e., $c\,Q \le \tau s(\tau,n)$ and ii) that speculators participate in the ICO, i.e., $u(s(\tau,n)) \ge 0$. Using the market clearing condition Lemma 1(ii), which ties token value τ_{eq} to sales $\min\{Q,D\}$, the firm's optimization problem can be formally written as:

$$\max_{\tau,\,n} \left\{\tau\, s(\tau,n) + \max_{Q} \;\left[(m-s(\tau,n))\,\frac{v}{m}\,\mathbb{E}[\min\{Q,D\}] - c\,Q\right]\right\} \tag{3}$$

subject to

$$\tau\, s(\tau,n) - c\,Q \ge 0, \quad \text{(ICO funds cover production costs)}$$

$$u(s(\tau,n)) \ge 0. \qquad \text{(speculators' participation constraint)}$$

Recall that $s(\tau,n)$ is an equilibrium quantity, and we will show later how it depends on the firm's decisions variables, τ and n, and on Q (which itself depends on s, and hence τ and n).

3 Analysis: Utility Tokens

In this section, we find the subgame perfect equilibrium using backward induction. We first consider (§3.1) the firm's last decision, the production quantity for fixed token price τ and ICO cap n, based on which we examine the speculators' equilibrium behavior (Appendix B.1). We then calculate the optimal token price τ^* and ICO cap n^* (§3.2). Lastly, we present and discuss the equilibrium results in §3.3.

[7] Note, if the product had salvage value, this value would need to be included in the market clearing condition.

3.1 Optimal Production Quantity

We first consider the firm's last decision—the production quantity $Q(\tau, n, s(\tau, n))$, for fixed token price τ and ICO cap n. Here and below, we drop when possible the fixed arguments τ and n to ease exposition.

Proposition 1. (Optimal Production Quantity)
For a fixed token price τ, ICO cap n and number of speculators s, the firm's optimal production quantity $Q^*(s)$ is as follows.

i) If $0 < s < m\left(1 - \frac{c}{v}\right)$, then $Q^*(s) = \min\left\{F^{-1}\left(1 - \frac{cm}{(m-s)v}\right), \frac{\tau s}{c}\right\}$.
ii) If $s = 0$ or $s \geq m\left(1 - \frac{c}{v}\right)$, then $Q^*(s) = 0$.

Part (i) of Proposition 1 shows that production can occur only if the number of speculators that purchased tokens in the ICO, is below a fraction $\left(1 - \frac{c}{v}\right)$ of all available tokens m. The first term inside the minimum operator, $F^{-1}\left(1 - \frac{cm}{(m-s)v}\right)$, is the unconstrained optimal production quantity; interestingly, this term decreases in s. The second term, $\frac{\tau s}{c}$, captures the firm's budget constraint, i.e., the production costs cannot exceed funds raised in the ICO, and this term is increasing in s.

Part (ii) of Proposition 1 shows that if more than a fraction $\left(1 - \frac{c}{v}\right)$ of all tokens have been sold in the ICO, the firm prefers not to use any of the funds raised for production, meaning, the firm "diverts" the money raised to its own pocket. We refer to this fraction as the firm's *misconduct fraction*,

$$1 - c/v. \tag{4}$$

Clearly, as the willingness-to-pay v increases relative to the production cost c, the misconduct fraction increases, making the abandonment of production less likely.

We emphasize that this analysis does not suggest all crypto startups are scammers that would run away with any amount. Rather, it provides an explanation for the loss of motivation or productivity of some well-funded startups based on pure profit maximization reasoning, due to moral hazard absent regulatory controls.

3.2 Optimal Token Price and ICO Cap

Given the optimal production quantity (§3.1) and speculators' equilibrium behavior (Appendix B.1), we now examine how the firm sets the profit-maximizing ICO token cap n^* and initial token price τ^*.

We show in Lemma 2 in Appendix B.1 that the number of speculators $s^*(\tau, n) \leq m\left(1 - \frac{c}{v}\right)$.

Given the speculators participating in the ICO buy 1 token each, we need not consider the case in which tokens $n > m\left(1-\frac{c}{v}\right)$. We will first find the token price $\tau^*(n)$ for a given token cap $n \le m\left(1-\frac{c}{v}\right)$ and then maximize profit over the token cap n. The following Proposition guarantees the existence of a nonzero equilibrium token price τ^*.

Proposition 2. (Conditions for ICO Success)

The ICO succeeds if and only if

i) (critical mass condition) the firm sells more than $\frac{mc}{v}$ tokens in the ICO and,

ii) (price-cost ratio requirement) customers have a high willingness-to-pay such that $v > 2c$.

Part (i) of Proposition 2 shows that the firm should not set the ICO cap too low. Speculators expect positive returns only when a critical mass of tokens, $\frac{mc}{v}$, are sold in the ICO. This quantity increases in the production cost and decreases in customer willingness-to-pay. Recall from Appendix B.1 that speculators would not invest more than the misconduct fraction. Combining these two results, we have that the ICO will only be successful when the misconduct fraction $m\left(1-\frac{c}{v}\right)$ is above the lower bound $\frac{mc}{v}$. This simplifies to the condition in Part (ii) of Proposition 2, $v > 2c$.

Next we find the optimal ICO token price $\tau^*(n)$ and the optimal ICO cap n^* assuming these two conditions are met. We show that for any fixed ICO cap n in the appropriate range ($n \in (\frac{mc}{v}, m\left(1-\frac{c}{v}\right))$), there exists a unique, positive and finite ICO token price $\tau^*(n)$ that maximizes (5) by extracting all utility from the speculators who participate strategically.

Given this result, we obtain a semi-closed-form solution of the optimal ICO cap n^*, and show that neither a small ICO cap that suppresses the production quantity nor a large cap that induces idle cash is profit-maximizing for the firm. The optimal ICO cap n^* allows the firm to raise just enough funds that can be credibly committed to production. We point interested readers to Appendix B.2 for detailed technical results.

3.3 The Equilibrium

Proposition 3. (Equilibrium Results)

i) If $v \le 2c$, then the ICO fails.

ii) If $v > 2c$, then there exists a unique equilibrium where

(a) the ICO cap n^* satisfies $n^* \in (\frac{mc}{v}, \frac{m}{2})$ and

$$\frac{v\,n^*}{c\,m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{cm}{(m-n^*)v})\right\}] = F^{-1}(1-\frac{cm}{(m-n^*)v}),$$

(b) the number of speculators is $s^* = n^*$,

(c) the ICO token price is $\tau^* = \frac{v}{m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{cm}{(m-n^*)v})\right\}]$,

(d) *the production quantity is* $Q^* = F^{-1}(1 - \frac{cm}{(m-n^*)v})$,

(e) *the token price in the market period equals the ICO token price, i.e.,* $\mathbb{E}[\tau_{eq}] = \tau^*$.

iii) *When* $v > 2c$*, the firm spends all funds raised through the ICO on production in equilibrium.*

Several results are of interest here, starting with the condition $v > 2c$, which implies that ICOs may be best suited for products with relatively high willingness-to-pay.[8]

Part (ii) summarizes the characteristics of the unique equilibrium when $v > 2c$. Part (a) links the ICO cap to operational and demand parameters. Although we do not have a closed-form expression for n^*, our model suggests that it is optimal for the firm to save a substantial portion of tokens (more than a half) to the market period. When more than $m/2$ tokens are sold, the firm raises more money than what is needed for the unconstrained optimal production quantity. The firm then produces at the unconstrained optimal level and, as a result, is left with some idle funds. However, these idle funds are gained at the expense of its share of the future revenue. As a result, the firm produces less and collects less money. We show in Appendix D that the decrease in money raised has a bigger effect on the firms final wealth, which results in non-optimal profit.

The rest of the equilibrium quantities depend on the optimal ICO cap n^*. Note that, since the total number of tokens available is kept constant, the ICO cap n^* is a proxy for the fraction of tokens sold during the ICO period. Part (b) shows that the ICO cap directly controls the number of speculators that will take part in the ICO. From parts (c) and (d), we can see that token prices and production quantity both decrease in the ICO cap, implying also a decrease of these quantities in the number of speculators. The latter interpretation may be more counterintuitive, but follows from the fact that ICO cap and speculator numbers are interchangeable. Note that, since the total number of tokens available is kept constant, the ICO cap n^* is a proxy for the fraction of tokens sold during the ICO period. Part (e) shows that in equilibrium, speculators' expected utility is zero because the expected market token price is equal to the ICO token price.

Part (iii) shows that the firm is incentivized to produce as much as possible in equilibrium. Note that we model an unregulated environment where the firm has the option to divert the funds raised (moral hazard), and the high margin condition prevents the firm from doing this. In fact, in the absence of moral hazard, first best can be achieved without the high margin condition. Part (iii) also implies that since production cost is fixed, the total funds raised $s^* \cdot \tau^*$ follows the same

[8]It is interesting to note this condition does not depend on demand characteristics. This is because this condition stems from the presence of moral hazard: it simply defines the cutoff between an ICO that will never be able to raise any cash (even when demand risk is low) to one that will raise some cash in equilibrium (epsilon or more) depending on demand risk, among other things. From that cutoff point onward, how successful the ICO will be (e.g. how much money it will raise) depends critically on demand characteristics.

trend as the equilibrium production quantity Q^*. We provide more insights on the equilibrium based on the numerical experiments in Appendix C.1.

Having analyzed the ICO equilibrium, we compare it to the first-best outcome so as to quantity agency costs. In this case, the first best refers to ICOs without frictions, i.e., ICOs with no cash diversion by the firm. While such "first-best" ICOs do not exist given the loose regulatory environment, by the Modigliani-Miller theorem, they are equivalent to a traditional newsvendor firm that invests its own money and faces no financial constraint.

Proposition 4. (ICO vs First Best)

i) A traditional newsvendor firm invests when $v > c$ whereas an ICO is only viable when $v > 2c$.

ii) A firm who finances through an ICO produces less than first best.

iii) A firm who finances through an ICO makes less profit than first best.

iv) In case of low demand realization, a traditional newsvendor risks loss whereas a firm who finances through an ICO always earns non-negative profit.

By Proposition 4, ICOs have the great advantage of being a low-risk means of financing for firms, but this comes at a cost of production quantity, profit and flexibility in terms of margin. We evaluate the extent of these benefits and inefficiencies numerically in Appendix C.1. Our numerical results show that in general, the production and profit gaps between the ICO firm and first best can reach up to 40% and up to 50%, respectively, but these gaps shrink when the market is bigger, more stable or (and) with a higher willingness-to-pay. Under the same market conditions, ICOs lead to lower profit variance, rendering firm profits less sensitive to demand uncertainty.

4 ICOs with Equity Tokens (STOs)

In this section, we consider a different type of ICO—one with equity, rather than utility, tokens (also referred to as STOs as mentioned). The fund-raising mechanism with equity tokens follows that with utility tokens (Figure 1) but with two main differences. First, the fundamental value of the equity tokens and the utility tokens are backed by the firm's future revenue and profit respectively. To see this, recall from Lemma 1 that the value of the utility tokens is equal to the worth of all products sold. The equity tokens, by definition, entitle the token holders to a pre-specified share of the firm's profit as long as the firm is profit-making. Second, the equity tokens have no utility purposes—in the market period, the firm sells its products for cash and distributes its profit among the equity token holders in proportion to their token holdings. As a result, the firm, unlike a utility-token-issuing firm, does not need to sell the remaining tokens (i.e., tokens unsold in the ICO period) in the market period.

Our analytical results identify two differences and two similarities between ICOs and STOs: 1) STOs are associated with lower agency costs; 2) STOs require a larger ICO cap to be successful; 3) both require the same high-margin condition; 4) both leave no arbitrage opportunities for speculators. We discuss the intuition behind these results below and leave the technical results to Appendix B.4.

1) Lower agency costs. Recall that the misconduct fraction with utility tokens is $1 - c/v$. In the case of equity tokens, the misconduct fraction is 1. This shows that as long as the firm does not sell out all the tokens during the ICO, i.e., $s \neq m$, it always produces some product if it raises money. Since $1 > 1 - c/v$, we argue that with equity tokens, the firm's incentives are better aligned with the speculators', making the firm less likely to divert cash from funds raised to its own pocket. In other words, STOs reduce moral hazard, thus having lower agency costs than utility ICOs.

2) Larger ICO cap. Consider a firm that aims to produce a certain quantity and could finance through issuing either utility tokens or equity tokens. By the nature of the two (revenue-sharing vs profit-sharing), we know that the market equilibrium price of the utility token will be higher than that of the equity token, i.e., $\tau_{eq} > \tau_{eq,e}$. We know by Proposition 5(i) (Appendix B.2) that for the utility tokens, the firm sets the ICO token price to be exactly equal to the expected market equilibrium token price, i.e., $\tau = \mathbb{E}[\tau_{eq}]$. For the equity tokens, since the speculators would only purchase the tokens when the ICO token price does not exceed the expected market equilibrium token value, we must have $\tau_e \leq \mathbb{E}[\tau_{eq,e}]$. Therefore, the optimal ICO price of the equity token, τ_e, must be less than that of the utility token, τ. As a result, to meet the same production goal, the firm will have to sell more equity tokens than utility tokens. Similar intuition and reasoning lead to a more stringent critical mass condition, which translates into a higher ICO cap.

3) Same high-margin condition. The function of the high-margin condition is to keep the feasible range of the ICO cap nonempty. By parts 1) and 2), with equity tokens, the fraction of tokens to be sold during the ICO has both a higher upper bound (misconduct fraction) and a higher lower bound (critical mass condition). As a result, the high-margin condition remains the same.

4) No arbitrage opportunities. In both ICOs and STOs, the ICO token price does not affect decisions or outcomes in the production or market periods. Therefore, the firm is able to set the ICO token price to the highest possible, leaving the speculators with zero expected profit.

We study the rest of the equilibrium results numerically in Appendix C.2. Through numerical experiments, we find that issuing equity tokens incentivizes the firm to produce more products, ceteris paribus. While good market conditions (high mean, low variance, high willingness-to-pay) reduce the extent of underproduction in both cases, they push the production level of the firm that issues equity tokens even closer to first best. This suggests that the first-best is almost achievable with equity tokens.

5 Extensions and Future Opportunities

5.1 Extensions

While our core model (ICO with utility tokens) is relatively basic, it is flexible enough to be extended in many ways to fit a variety of practical situations.

Risk of Product Failure Motivated by the Sirin Labs example discussed in the introduction, our base model assumes that the firm is able to successfully produce its product when it incurs the necessary production cost. In Appendix B.5 we relax this assumption, and introduce the risk of production failure. We find that the firm's optimal strategy can qualitatively change depending on the amount of production risk and customer willingness-to-pay. Recall that in Proposition 3 (iii), we show that when there is no risk, the firm invests all money raised into production. In the presence of production risk, in many cases, we find that the firm spends only part of the ICO funds raised on production, and saves the rest. Such practice guarantees that the firm ends up with positive final wealth even if production fails. We refer interested readers to Appendices B.5 and C.3 for more details.

Speculators with Outside Investment Options We can account for the existence of other investment options (e.g., a savings account) by adding a generic investment option that returns $k > 0$ dollars per dollar investment. We show that a higher return of the outside option makes ICOs harder to succeed and creates a wedge between the token price in the ICO and the expected token price post ICO. Moreover, in equilibrium, the expected return of the tokens is equal to that of the outside option. More details can be found in Appendices B.6 and C.4.

5.2 Future Opportunities

As one of the first papers to study the implications of ICOs for operations management, the model we develop has of course some limitations that could represent interesting research opportunities.

For example, it could be interesting to study the multi-period production setting, which could also involve issues of token resale and inflation control. In practice, many projects keep a portion of ICO funds and/or tokens to maintain price stability in the future and protect against negative shocks. Moreover, some entrepreneurs need initial funds for the design and preparation of an ICO, which requires a different ICO design or even other financing solutions. Another interesting direction would be to expand the firm's decision space in terms of where it can spend the raised funds, to include other business functions such as marketing, human capital, etc.

Lastly, several assumptions in our model could be relaxed to capture more realistic settings. For instance, the tokens could be used for purposes other than to purchase physical goods; customer willingness-to-pay and demand could be affected by quite a few factors that we do not capture, including network effects; the success of the ICO could be informative about future demand in a multi-period setting; investors could have heterogeneous beliefs about product quality; customers could have different valuations for the product; firms, investors and/or customers could be risk averse or risk seeking, etc.

We believe these to be promising directions for future research.

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Appendix

A Additional Discussions and Results

A.1 Utility Tokens and the Token Buyers

In this section, we elaborate on two important features of tokens and the role of the token buyers (the speculators and the customers).

First, tokens play a dual role: as of today, most tokens in the market have been considered as both utility and security.[9] The security aspect results from the tradable feature of the tokens. The utility aspect comes from the fact that the fundamental value of these tokens lies in the economic value of the products or services that they are redeemable for. However, most projects do not have any products at the time of the ICO. In 2017, for instance, 87% of ICOs did not yet have a running product (CryptoGlobe 2018). To capture these features, we model tokens that start out as pure securities and only after product launch become utility tokens. Such tokens appeal to two groups of token buyers: those who see tokens as securities purchase the tokens in the ICO period (before product launch),[10] whereas those who wish to consume the products buy tokens in the market period (after product launch). Therefore, we refer to the token buyers in the ICO period and those in the market period as *speculators* and *customers* respectively.

The second feature is that the tokens issued by the firm can only be redeemed on the firm's own platform and are the only viable method of payment for the its products. By restricting the method of payment, the firm ties the value of the tokens to the economic value of the products. This, together with the existence of a secondary market to trade the tokens, incentivize speculators to purchase tokens in the ICO, even if they are not interested in subsequently consuming the product themselves.

At the same time, the fact that the tokens have no use on other platforms has a few implications. First, it means that the token value solely depends on the consumption of products of this particular platform. Second, after the firm ends production, the speculators have no reason to hold the tokens and the customers do not buy more tokens than needed. Third, redeemed tokens retain no value if no further production is planned. Last, since we only consider one round of production, this suggests that the tokens are for one-time use only and the firm cannot resell the redeemed tokens

[9] The regulatory environment is still uncertain but efforts are being made to pass bills that would distinguish tokens from securities like stocks (Khatri 2019).

[10] Technically, those who see tokens as securities may purchase tokens whenever they feel optimistic about the potential return. However, we model a firm that plans one round of production and product sale and the market token price in the market period is an equilibrium quantity that does not change during that period. Therefore, it only makes sense for this group of token buyers to come in the ICO period.

for more cash.

A.2 ICOs vs Other Early-Stage Financing Methods

In this section, we summarize important structural differences between ICOs and their alternatives in Table 1. We also discuss in more detail two particular aspects that distinguish ICOs from other early-stage financing methods: the existence of a secondary market and the issuance of tokens.

Table 1: Comparison of Early-Stage Financing Methods. (A checkmark ✓ indicates the feature is prominent, while ✗ indicates it is of second-order or non-existent. The dual notation ✓✗ indicates that the feature may or may not be of first order, depending on circumstance.)

	Bank	**VC**	**Crowdfunding**		**Coin Offering**	
			Reward	Equity	Utility	Equity
Upside through Profit Sharing	✗	✓	✗	✓	✗	✓
Upside through Revenue Sharing	✗	✗	✗	✗	✓	✗
Downside Demand Risk Sharing	✓	✓	✗	✓	✓	✓
Heavily Regulated	✓	✓	✗	✓	✗	✓✗
Voting/Control Rights	✓✗	✓	✗	✗	✗	✓✗
Funds from Retail Speculators	✗	✗	✗	✓	✓	✓
Funds from Retail Consumers	✗	✗	✓	✓	✓	✓
Secondary Trading	✗	✗	✗	✗	✓	✓

Table 1 contains a large amount of information, and we recommend it be read through bilateral column comparisons. The high-level takeaway from the table is that ICOs, be it utility or equity offerings, differ from each of the other alternative forms of financing in at least one crucial dimension (and more often than not, in several dimensions). We highlight two of these aspects next.

1. Implications of the existence of a secondary market

ICOs differ from all other financing methods by their reliance on a secondary market for the tokens. This difference has two important implications.

1) **Mitigation of Moral hazard.**

   The alternative financing methods listed in Table 1 address moral hazard in different ways. Banks, for instance, use interest rates and covenants (Iancu et al. 2016) and/or leverage collateral. VC firms directly monitor the progress of the funded company and invest in stages to keep the company under control (Cherif and Elouaer (2008); Wang and Zhou (2004)). In crowdfunding, moral hazard is often left unadressed, though more recently, some platforms like Indiegogo have started to use escrow accounts to mitigate it (Belavina et al. 2019).

   In the case of ICOs, there typically exists no third party between the fundraising firm and its investors. Instead, moral hazard is addressed, among other things, via the existence of a peer-to-peer secondary market for the tokens. To see this, consider that in our model, the fraction of tokens sold during the ICO, in equilibrium, is below the misconduct fraction, motivating the firm to produce at a level that leads to the highest expected equilibrium token price. Thus, in contrast to the alternative financing methods mentioned above, ICOs fight moral hazard without the need for an intermediary.

2) **Nature of Investors.**

The ICO secondary market is a peer-to-peer market that allows all token owners to jointly sell the tokens to those who desire them. As discussed in Appendix A.1, this suggests that the investors (speculators) do not have to be the consumers of the firms products. In contrast, entrepreneurs running traditional crowdfunding campaigns (e.g., on Kickstarter), pre-sell their products directly to early adopting customers during the fundraising stage. This implies that the majority, if not all, of the backers in crowdfunding campaigns are the actual product consumers. Given the different nature of investors, it is reasonable to argue that ICOs have access to a larger investor pool than the crowdfunding projects. Indeed, an average ICO project in 2018 was able to raise $11.52 million (Cointelegraph 2019), which is closer to the average VC deal value in the same year ($14.6 million) (PitchBook 2019) and far exceeded the crowdfunding average ($10k) (Kickstarter 2019).

2. Implications of the issuance of tokens

While both ICOs and crowdfunding raise funds through retail investors, the issuance of tokens further differentiates ICOs from crowdfunding. Our model shows that the utility tokens allow revenue sharing and the equity tokens allow profit sharing among all token holders. In addition, the tokens dilute the impact of future demand on the firm by allowing the firm to disperse the downside risks of low demand realization among the investors. On the contrary, the backers of a crowdfunding campaign do not share such risks because a low demand in the crowdfunding aftermarket would only hurt the firm's profit.

A.3 Example: Honeypod Whitepaper

Honeypod (Honeypod 2018) aims to produce a hardware that serves as the main hub that interconnects various devices and provides traffic filtering. The company claims that they have mature products that are ready for mass production before token crowdsale.

Parameters captured by our model include

1. Hard cap ($m = 200,000,000$).
2. ICO sales cap/soft cap ($n = 40,000,000$).

3. Fixed token price of during public token sale ($\tau = \$0.05$).
4. Customers' willingness to pay ($v = \$99$).
5. Manufacturing cost ($c = \$32$).
6. Production quantity over 12 months ($Q = 50,000$).

Parameters not captured by our model include

1. Four tiers of fixed token prices during private token sale ($\$0.02, \$0.025, \$0.03, \$0.035$).
2. Other use of funds from the token sale (e.g. 25% on maintenance, R&D).

Parameters in our model that are not mentioned in the white paper include

1. Aggregate demand (D).

B Technical Results

B.1 Equilibrium Number of Speculators and Participation Constraint

Having derived the firm's optimal production quantity for a given ICO design τ, n, we next examine the implications on speculators.

Lemma 2. (Speculator Equilibrium Properties) *Given initial token price τ and the sales cap n,*

i) The number of speculators who purchase tokens is $s^*(\tau, n) = n \cdot \mathbb{1}_{\{u(n)\geq 0\}}$,

ii) $s^*(\tau, n) \in \left[0, m\left(1 - \frac{c}{v}\right)\right]$, i.e., there is no fund diversion in equilibrium,

iii) Define $s_0(\tau) = \max\{0 < s \leq m : u(s) = 0\}$. If $s_0(\tau)$ exists, $s_0(\tau) < m\left(1 - \frac{c}{v}\right)$ and $u(s) < 0$ for all $s > s_0(\tau)$.

Lemma 2, part i) is a compact way to write that in equilibrium, the number of speculators purchasing tickets is equal to the ICO cap, as long as speculators' participation constraint is satisfied. This is because all speculators have the same expected profit, and hence, either n speculators will purchase tokens (if this expected profit is ≥ 0), or none will. Note, this result holds for sequential arrivals as well.

Lemma 2, part ii) defines a lower and upper bound on the number of speculators that arises in equilibrium. The lower bound is trivial. The upper bound is a consequence of the firm's misconduct threshold derived in Proposition 1, and captures the fact that in any equilibrium, speculators strategically prevent their funds from being diverted.

Lemma 2, part iii) is a necessary technical condition ensuring speculator participation constraint holds, and hence, the success of the ICO. In the Sections 3.2 and B.2, we show that the existence of $s_0(\tau)$ depends on τ, which in turn depends on n, and discuss the implications.

B.2 Optimal Token Price and ICO Cap

Given the optimal production quantity and speculators' equilibrium behavior, we now examine how the firm sets the profit-maximizing ICO token cap n^* and initial token price τ^*.

From Lemma 2, the number of speculators $s^*(\tau, n) \le m\left(1 - \frac{c}{v}\right)$, and given speculators participating in the ICO buy 1 token each, we need not consider the case in which tokens $n > m\left(1 - \frac{c}{v}\right)$. We will first find the token price $\tau^*(n)$ for a given token cap $n \le m\left(1 - \frac{c}{v}\right)$ and then maximize profit over the token cap n.

For a fixed n, the platform's optimization problem (3) can be written as a maximization problem over τ subject to speculators' participation constraint. In particular, the optimization problem is

$$\max_{\tau \ge 0} \Pi = \tau(n)\, s^*(\tau, n) - c\, Q^*(s^*(\tau, n)) + (m - s^*(\tau, n))\, \mathbb{E}[\tau_{eq}(s^*(\tau, n))], \tag{5}$$

subject to $u(s^*(\tau, n)) \ge 0$ and $Q^*(s^*(\tau, n)) = \min\left\{F^{-1}\left(1 - \frac{cm}{(m - s^*(\tau, n))v}\right), \frac{\tau\, s^*(\tau, n)}{c}\right\}$ (from Proposition 1). Proposition 2 (see Section 3.2) guarantees the existence of a nonzero equilibrium token price τ^*.

Next we find the optimal ICO token price $\tau^*(n)$ assuming the two conditions in Proposition 2 are met. Before stating the proposition, we impose an additional technical condition on the demand distribution to guarantee equilibrium uniqueness[11]: $\frac{f'(F^{-1}(y))}{(f(F^{-1}(y)))^2} > -\frac{3v}{c} \cdot k \cdot \frac{1-k}{2k-1}$ where $k = 1 - \frac{c}{(1-y)v}$ and $y \in [0, 1 - \frac{2c}{v})$.

Proposition 5. (Optimal ICO Token Price)

When $v > 2c$,

i) For a given $n \in (\frac{m\,c}{v}, m\left(1 - \frac{c}{v}\right)]$, there exists a finite positive $\tau^*(n)$ uniquely determined by $u(s^*(\tau^*(n))) = 0$.

ii) There exists a unique $\hat{n} \in (\frac{m\,c}{v}, \frac{m}{2})$ such that

- *for $n \in \left[\frac{m\,c}{v}, \hat{n}\right)$, $\tau^*(n)$ is the unique solution of $\tau^*(n) = \frac{v}{m}\,\mathbb{E}\left[\min\left\{D, \frac{\tau^*(n)\,n}{c}\right\}\right]$;*
- *for $n \in \left[\hat{n}, m\left(1 - \frac{c}{v}\right)\right]$, $\tau^*(n) = \frac{v}{m}\,\mathbb{E}\left[\min\left\{D, F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)\right\}\right]$.*

Part (i) of Proposition 5 shows that when the price-cost ratio is high enough, for any fixed ICO cap n in the appropriate range as suggested by Proposition 2 (i), there exists a unique, positive and finite ICO token price $\tau^*(n)$ that maximizes (5) by extracting all utility from the speculators who participate strategically according to Lemma 2. By (1), this implies that the expected equilibrium

[11] One can readily check analytically or numerically) that this sufficient condition is generally satisfied for some common distributions such as uniform and normal. All numerical results presented in the paper satisfy this condition.

token price in the market period is equal to the optimal ICO token price, i.e., $\mathbb{E}[\tau_{eq}(s(\tau^*(n), n)] = \tau^*(n)$. We then solve $u(s^*(\tau^*(n))) = 0$ using Lemma 1 (ii) and Proposition 1 (i) and obtain part (ii) of Proposition 5. Recall that the term $\frac{\tau^*(n)\, n}{c}$ reflects the budget constraint and $F^{-1}\left(1 - \frac{c\, m}{(m-n)v}\right)$ is the constrained optimal production quantity. Therefore, part (ii) of Proposition 5 suggests that the firm, upon setting the optimal ICO token price, spends all funds raised on production when the ICO cap n is small but produces an optimal quantity without using all the funds when n is large or $\frac{n}{m}$ is closer to the misconduct fraction.

Knowing $\tau^*(n)$, $s^*(\tau^*(n), n)$ and $Q^*(s^*(\tau^*(n), n))$, the firm's optimization problem reduces to a maximization problem over the ICO cap n given by

$$\begin{aligned} \max_{\frac{m\,c}{v} < n \leq m\left(1-\frac{c}{v}\right)} \Pi \;&=\; \tau^*(n)\, s^*(\tau^*(n), n) \,-\, c\, Q^*(s^*(\tau^*(n), n)) \\ &\quad + (m - s^*(\tau^*(n), n))\, \mathbb{E}[\tau_{eq}(s^*(\tau^*(n), n))] \end{aligned} \tag{6}$$

where $s^*(\tau(n), n) = n$, $Q^*(s^*(\tau(n), n)) = \min\left\{F^{-1}(1 - \frac{cm}{(m-n)v}), \frac{\tau^*(n)\, n}{c}\right\}$ and $\tau^*(n)$ is given by Proposition 5 part ii).

This leads to the following result.

Proposition 6. (Equilibrium ICO Cap) *When $v > 2c$, the unique optimal ICO cap $n^* \in (\frac{m\,c}{v}, \frac{m}{2})$ equals the threshold $\hat{n}$ in Proposition 5 ii), and is the solution to the following equation:*

$$\frac{v\, n^*}{c\, m}\, \mathbb{E}\left[\min\left\{D, F^{-1}(1 - \frac{c\, m}{(m-n^*)v})\right\}\right] = F^{-1}\left(1 - \frac{c\, m}{(m-n^*)v}\right).$$

Proposition 6 tells us that neither a small ICO cap that suppresses the production quantity nor a large cap that induces idle cash is profit-maximizing for the firm. The optimal ICO cap n^* allows the firm to raise just enough funds that can be credibly committed to production, and here we provide a semi-closed-form solution of n^*.

B.3 Sequential Arrival of Speculators

In this section, we assume that the z speculators arrive sequentially during the ICO period and observe the number of tokens sold before their arrival, rather than showing up simultaneously. Tokens are sold on a first-come, first-served basis and each speculator buys either zero or one token based on the expected profit of their purchase. We will show that while this alternative assumption on the speculators' arrival changes one of the intermediate results, it leads to the same equilibrium results as in the rest of the paper.

Suppose the first s speculators will buy one token each. Then anyone who arrives later than the $s-th$ speculator will not buy any token and thus obtains zero utility. In this section, we focus on the earliest s arrivers. The expected profit of such a speculator given there s tokens will be sold by the end of the ICO is given by

$$u(s) = \Delta(s)\mathbb{1}_{\{s>0\}}, \tag{7}$$

where $\Delta(s)$, by (1), Lemma 1 and Proposition 1, is

$$\Delta(s) = \frac{v}{m}\,\mathbb{E}\left[\min\left\{D, F^{-1}\left(1-\frac{cm}{(m-s)v}\right), \frac{\tau\, s}{c}\right\}\cdot \mathbb{1}_{\left\{s<m\left(1-\frac{c}{v}\right)\right\}}\right] - \tau. \tag{8}$$

The participation constraint requires that $u(s) \geq 0$. From (8) we immediately know that the equilibrium number of speculators will never be $m\left(1-\frac{c}{v}\right)$ or beyond because $u(s) = -\tau < 0$ for $s \geq m\left(1-\frac{c}{v}\right)$. Therefore, the speculators who arrive sequentially would collectively buy under the misconduct fraction.

Since $u(s)$ and $\Delta(s)$ have the same sign for $s > 0$, Lemma 2 part iii) still holds. Lemma 2 part iii) tells us that when the speculators arrive sequentially, there will be exactly $s_0(\tau)$ speculators without the sales cap n. However, note that $s_0(\tau)$ is not necessarily the utility-maximizing s because the early speculators cannot stop those who arrive later from buying more tokens unless it is no longer profitable to do so.

So far there are two upper bounds of the equilibrium number of speculators s^*: the sales cap, n, and $s_0(\tau)$[12]. We express s^* in terms of these two upper bounds in the following proposition.

Proposition 7. (Equilibrium Number of Sequentially Arriving Speculators)
Given initial token price τ and the sales cap n, the equilibrium number of speculators is given by

$$s^*(\tau, n) = \min\{s_0(\tau), n\} \tag{9}$$

provided that $s_0(\tau)$ exists and

$$u(s^*) \geq 0. \tag{10}$$

If $s_0(\tau)$ does not exist or $u(\min\{s_0(\tau), n\}) < 0$, then there will be no speculators and thus ICO fails.

Note that the expression of $u(s)$ with simultaneous arrivals given by (1) and that with sequential

[12]In this section, we assume that $s_0(\tau)$ exists because its existence is necessary for $u(s^*) \geq 0$ for some $s > 0$. We show that the existence of $s_0(\tau)$ depends on both τ and n and discuss the conditions (the critical mass condition and a high willingness-to-pay) in Section 3.2.

arrivals given by (7) have the same sign, albeit differing by a scale of s/z for $s > 0$. Since the magnitude of the speculators' utility does not affect their purchase decision or the firm's profit, Propositions 2 - 4 and Proposition 3 (iii) hold for both arrival assumptions. Details can be found in Appendix D.

Moreover, following Proposition 3, we can show that setting a sales cap is not needed when the customers observe their arrival sequence.

Corollary 1. *When $v > 2c$, in equilibrium we have $n^* = s_0(\tau^*(n^*))$.*

By Corollary 1, the optimal ICO sales cap is equal to the equilibrium number of speculators who would participate even when the cap is unannounced. Therefore, to reach the target level of token sales n^* that eventually induces maximum expected profit, it suffices to set the ICO token price to be $\tau^*(n^*)$.

B.4 Equity Tokens

This section contains the technical analysis for ICOs with equity tokens. By definition, in the market period, the realized value of each equity token is

$$\tau_{eq,e} = \frac{1}{m} \cdot (v \min\{D, Q_e\} - c\,Q_e)^+ . \tag{11}$$

The firm maximizes its expected dollar-denominated wealth at the end of the market period, denoted by Π_e, which consists of three terms: i) the total funds raised during the ICO, $\tau_e\, s(\tau_e, n_e)$, plus ii) the expected total profit, $v\,\mathbb{E}\,[\min\{D, Q\}] - c\,Q$, minus iii) total payout to other token holders, $s(\tau_e, n_e)\,\mathbb{E}\,[\tau_{eq,e}]$. The objective function is as follows.

$$\max_{\tau_e, n_e} \left\{ \tau_e\, s(\tau_e, n_e) + \max_{Q_e} \left\{ (v\,\mathbb{E}\,[\min\{D, Q_e\}] - c\,Q_e) - \frac{s(\tau_e, n_e)}{m}\,\mathbb{E}\,[v \min\{D, Q_e\} - c\,Q_e]^+ \right\} \right\} \tag{12}$$

subject to

$\tau_e\, s(\tau_e, n_e) - c\,Q_e \geq 0$, (ICO funds cover production costs)

$u(s(\tau_e, n_e)) \geq 0$. (speculators' participation constraint)

Again, we will find the subgame perfect equilibrium using backward induction.

First, we consider the optimal production quantity Q_e^* given fixed token price τ_e and ICO cap n_e. Let $Q_u^*(s)$ denote the optimal production quantity unconstrained by the budget.

Proposition 8. (Optimal Production Quantity with Equity Tokens)

For a fixed token price τ_e, ICO cap n_e and number of speculators $s \in (0, m)$, the firm's optimal production quantity is $Q_e^*(s) = \min\left\{Q_u^*(s), \frac{\tau_e\, s}{c}\right\}$, where $Q_u^*(s) > 0$ is the unique solution of

$$(m - s)[(1 - F(Q_u^*(s))\, v - c] = s\, c\, F\left(\frac{c}{v}\, Q_u^*(s)\right). \tag{13}$$

We show in the proof of Proposition 8 that $Q_u^*(s)$ decreases monotonically in s and the firm, ignoring the budget constraint, would produce at the first best production when $s = 0$ as $Q_u^*(0) = F^{-1}\left(\frac{v-c}{v}\right)$. Therefore, for any positive number of speculators, the firm produces less than the first-best quantity. The other boundary case is $Q_u^*(m) = 0$. Since $Q_e^*(m) = \min\left\{Q_u^*(m), \frac{\tau_e\, m}{c}\right\} = \min\left\{0, \frac{\tau_e\, m}{c}\right\} = 0$, the firm produces nothing when $s = m$. Therefore, in the case of equity tokens, the misconduct fraction is 1.

At this point, we make a regularity assumption on the demand distribution[13]: $f(x) < a^2 \cdot f(ax)$ for $a > 2$. Using the result of Proposition 8, we show next that successful ICOs with equity tokens require a larger fraction of the tokens to be sold during the ICO than those with utility tokens.

Proposition 9. (Conditions for ICO Success with Equity Tokens)

An ICO that issues equity tokens succeeds if and only if

- *i) (critical mass condition) the firm sells more than $\frac{c}{v-c}\, m$ tokens in the ICO and,*
- *ii) (price-cost ratio requirement) customers have a high willingness-to-pay such that $v > 2c$.*

Recall from Proposition 2 part (i) that with utility tokens, the minimum number of tokens needed for production is $\frac{c}{v}\, m$. Since $\frac{c}{v-c}\, m > \frac{c}{v}\, m$, part (i) suggests a more stringent critical mass condition for equity tokens. Following part (i) and Proposition 8 that the firm should not sell all of its equity tokens, we need $\frac{c}{v-c}\, m < m$ for the existence of feasible n, which leads to part (ii). Comparing with Proposition 2 part (ii), we see that the price-cost ratio requirement is the same for both types of tokens. Therefore, while intuitively the equity tokens put an emphasis on "profit" by definition, they do not require a higher or lower profit margin of the product than the revenue-sharing utility tokens.

Lastly, we show that when the two conditions given by Proposition 9 are met, the firm sets the ICO token price such that the speculators' expected utility is zero—a similar result to Proposition 5(i).

[13]This assumption is satisfied by distributions that do not contain sharp peaks such as uniform distributions and most normal distributions.

Proposition 10. (Optimal ICO Equity Token Price)

When $v > 2c$, for a given $n_e \in (\frac{c}{v-c}\, m,\, m)$, there exists a finite positive $\tau_e^*(n_e)$ uniquely determined by $u(s^*(\tau_e^*(n_e))) = 0$.

B.5 Extension: Risk of Production Failure

In this section, we provide technical details for the extension with risks of production failure in Section 5. Let $\alpha \in (0, 1]$ denote the probability that the firm's technology leads to successful production and suppose that the value of α is common knowledge. The firm either successfully produces the decided quantity or ends up with zero acceptable products. We also assume that the firm finds out whether production has been successful at the end of the production period, after it has paid the necessary production cost for the decided quantity. In other words, the production cost is sunk regardless of the outcome of production.

Given such risks, the equilibrium token price is given by $\tau_{eq} = (1-\alpha) \cdot \frac{v}{m} \min\{0, D\} + \alpha \cdot \frac{v}{m} \min\{Q, D\}$, and thus $\mathbb{E}[\tau_{eq}] = \alpha \cdot \frac{v}{m} \mathbb{E}[\min\{Q, D\}]$. The firm solves a modified objective function

$$\max_{\tau,\, n} \left\{ \tau\, s(\tau, n) + \max_{Q} \ \left[\alpha\, (m - s(\tau, n))\, \frac{v}{m}\, \mathbb{E}[\min\{Q, D\}] - c\, Q \right] \right\} \tag{14}$$

subject to

$$\tau\, s(\tau, n) - c\, Q \geq 0, \quad \text{(ICO funds cover production costs)}$$

$$u(s(\tau, n)) \geq 0. \qquad \text{(speculators' participation constraint)}$$

We show next that riskier production intensifies the moral hazard problem.

Proposition 11. (Optimal Production Quantity under Risks of Production Failure)

Suppose the firm's production is successful with probability $\alpha \in (0, 1]$. For a fixed token price τ, ICO cap n and number of speculators s, the firm's optimal production quantity $Q^*(s)$ is as follows.

i) If $0 < s < m(1 - \frac{c}{\alpha v})$, then $Q^*(s) = \min\left\{ F^{-1}\left(1 - \frac{cm}{\alpha(m-s)v}\right), \frac{\tau\, s}{c} \right\}$.

ii) If $s = 0$ or $s \geq m(1 - \frac{c}{\alpha v})$, then $Q^*(s) = 0$.

Proposition 11 shows that given the same ICO token price and ICO cap, a lower success probability leads to lower production quantity. The firm is also more likely to give up production and run away when α is smaller because the misconduct fraction, $1 - \frac{c}{\alpha v}$, is lower. As a result, we show in Proposition 12 that ICOs are harder to succeed under greater production risks.

Proposition 12. (Conditions for ICO Success under Risks of Production Failure)

Suppose the firm's production is successful with probability $\alpha \in (0,1]$. Then, the ICO succeeds if and only if

i) (critical mass condition) the firm sells more than $\frac{m\,c}{\alpha\,v}$ tokens in the ICO and,

ii) (price-cost ratio requirement) customers have a high willingness-to-pay such that $v > \frac{2c}{\alpha}$.

B.6 Extension: Speculators with Outside Investment Options

In this section, we provide technical details for the extension with outisde investment options in Section 5. Suppose there exists a generic outside investment option that returns $k > 0$ dollars per dollar investment. The outside option provides a new reference point when the speculators evaluate their ICO return. Let $\Delta_i(s)$ denote the expected profit improvement by investing in an ICO with utility tokens. Then,

$$\Delta_i(s) = \mathbb{E}[\tau_{eq}(s)] - \tau - \tau\, k = \mathbb{E}[\tau_{eq}(s)] - (k+1)\,\tau, \tag{15}$$

and the speculators expected utility is $u(s) = \frac{s}{z}\Delta_i(s)$. The firm solves the same objective function as in (3). Therefore, the misconduct fraction is unaffected by the presence of the outside option, and the optimal production quantity in the subgame still follows that in Proposition 1. However, we show below that a higher return of the outside option makes ICOs harder to succeed as it leads to more stringent success conditions.

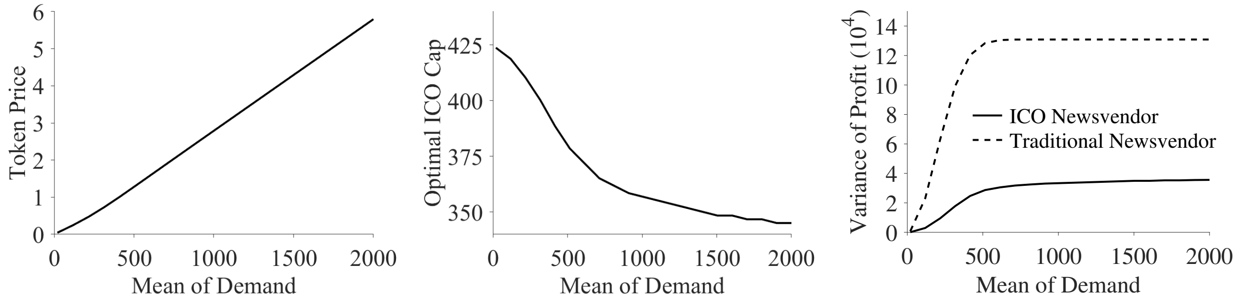
Proposition 13. (Conditions for ICO Success with an Outside Investment Option)
In the presence of an outside investment option with return k per dollar invested, the ICO succeeds if and only if

i) (critical mass condition) the firm sells more than $(1+k)\frac{m\,c}{v}$ tokens in the ICO and,

ii) (price-cost ratio requirement) customers have a high willingness-to-pay such that $v > (2+k)c$.

Next, we show that the optimal ICO token price makes the expected return of the tokens equal to that of the outside option.

Proposition 14. (Optimal ICO Token Price with an Outside Investment Option)
When $v > (2+k)\,c$, For a given $n \in \left((1+k)\frac{m\,c}{v}, m\left(1-\frac{c}{v}\right)\right)$, there exists a finite positive $\tau^*(n)$ uniquely determined by $u(s^*(\tau^*(n))) = 0$.



(a) Token Price vs Mean (b) ICO Cap vs Mean (c) Variance of Profit vs Mean

Figure 2: Impact of Mean of Demand

C Numerical Experiments

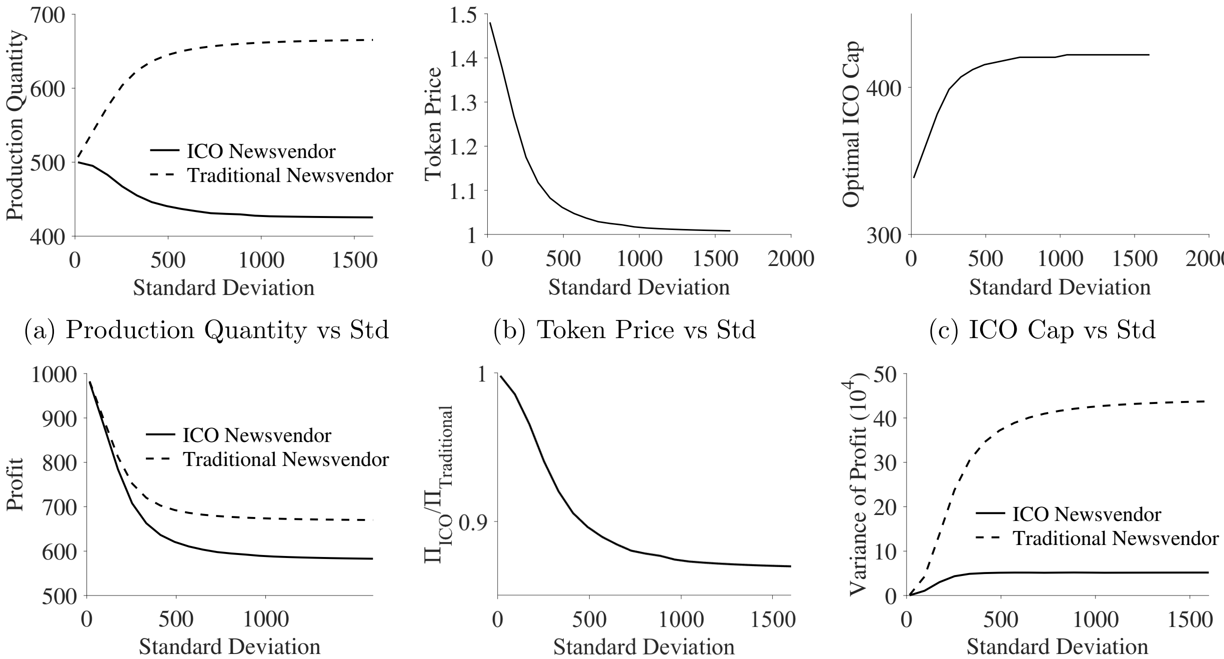
C.1 Numerical Experiments: ICOs with Utility Tokens

In this section, we provide a comparative-statics analysis through numerical experiments[14]. In particular, we focus on the impact of the mean and variance of demand and customers' willingness-to-pay.

When demand of the product is higher on average, the tokens, with a fixed supply, become more valuable (Figure 2 (a)). Therefore, the token price increases, allowing the firm to raise more capital in the ICO and produce a larger quantity to meet the demand (which can be readily checked). Note that the increase in product demand does not mean higher participation of speculators—in fact, we find the firm is incentivized to put a stricter cap on the ICO token sales and save more tokens to the secondary offering round (Figure 2 (b)). This is consistent with the inverse relationship between the cap n^* and the other equilibrium quantities including Q^* and τ suggested by Proposition 3. Interestingly, the effect of the appreciation in the token price dominates that of the reduction in the proportion of tokens sold in the ICO.

We see different trends with respect to demand variability of the two firms—when demand fluctuates more, the ICO newsvendor reduces production whereas the traditional newsvendor may stock up (Figure 3 (a)). Such distinction could well come from the fact that higher uncertainty in the market adversely affects speculators' confidence in the token, driving the token price down (Figure 3 (b)). When facing high demand variability, it is also in the firm's best interest to sell more tokens in the ICO (Figure 3 (c)). However, as suggested by Proposition 3, the firm, to

[14]In all of our numerical experiments throughout the paper, demand follows a truncated normal distribution distributed with mean μ, standard deviation σ, lower bound 1, upper bound 2μ. By default, the parameters are assigned values $\mu = 500$, $\sigma = 166$, $m = 1000$, $c = 1$ and $v = 3$. Note that while we don't use the parameters in the Honeypod example, the default price-cost ratio in our numerical experiments is very close to Honeypod's (3 vs 99/32).



(a) Production Quantity vs Std (b) Token Price vs Std (c) ICO Cap vs Std

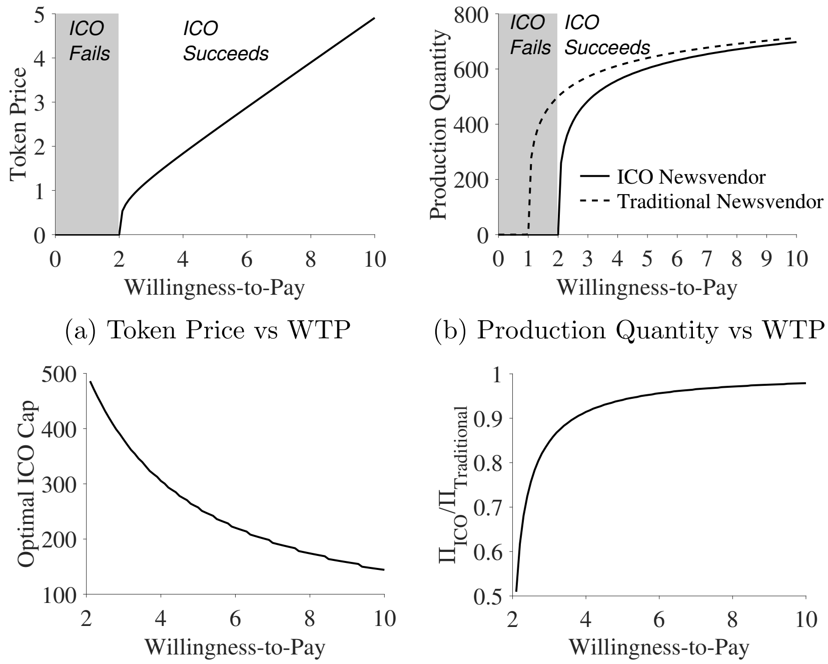
(d) Profit vs Std (e) Profit Ratio vs Std (f) Variance of Profit vs Std

Figure 3: Impact of Standard Deviation (Std) of Demand

guarantee the success of the ICO, needs to save at least half of all tokens for the market period. As a result, the decrease in the token price has a dominating effect on the funds raised. We find that greater demand uncertainty hurts both the firm's ability to raise capital (Figure 3 (a)) and its profit (Figure 3 (d)). Moreover, the profit gap between ICO financing and first best widens (Figure 3 (e)) as demand variability increases, suggesting that ICOs are better suited for products with a more predictable or stable market.

Similar to a higher mean demand, a higher willingness-to-pay boosts the token value (Figure 4 (a)) and allows the firm to raise more funds (Figure 4 (b)) in the ICO while saving a larger fraction of tokens for the secondary offering (Figure 4 (c)). However, the rate of increase in funds raised due to a higher v decreases in v whereas that due to a higher mean demand is almost constant. This is because the equilibrium cap on ICO token sales n decreases drastically in v and the reason the firm is motivated to save that many tokens for the secondary offering is that the firm mainly takes advantage of the higher profit margin rather than the higher sales volume. Moreover, while a higher willingness-to-pay incentivizes both the traditional newsvendor and the ICO newsvendor to produce more, it reduces the extent of underproduction by the ICO newsvendor (Figure 4 (b)).

Lastly, it can be readily checked that either a larger demand or a higher willingness-to-pay lead to a higher expected profit and reduce the profit gap between ICO financing and the first best



(a) Token Price vs WTP (b) Production Quantity vs WTP

(c) ICO Cap vs WTP (d) Profit Ratio vs WTP

Figure 4: Impact of Willingness-to-Pay

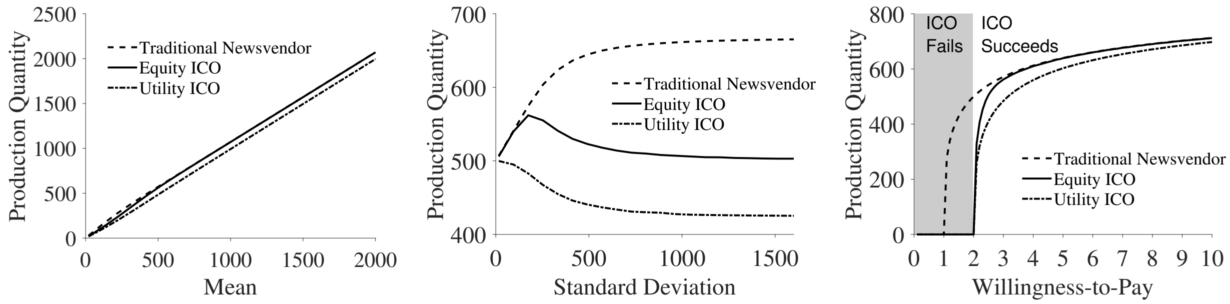
(Figure 4 (d)). Our numerical results also show that financing through an ICO leads to a lower variance of the expected profits (Figure 2 (c), 3 (f)).

C.2 Numerical Experiments: ICOs with Equity Tokens

In Sections 4 and B.4, we show analytically that ICOs with either type of token lead to underproduction. Through numerical experiments, we find that issuing equity tokens incentivizes the firm to produce more products (Figure 5), ceteris paribus. While good market conditions (high mean, low variance, high willingness-to-pay) reduce the extent of underproduction in both cases, they push the production level of the firm that issues equity tokens even closer to that of a traditional newsvendor. This suggests that the first-best is almost achievable with equity tokens.

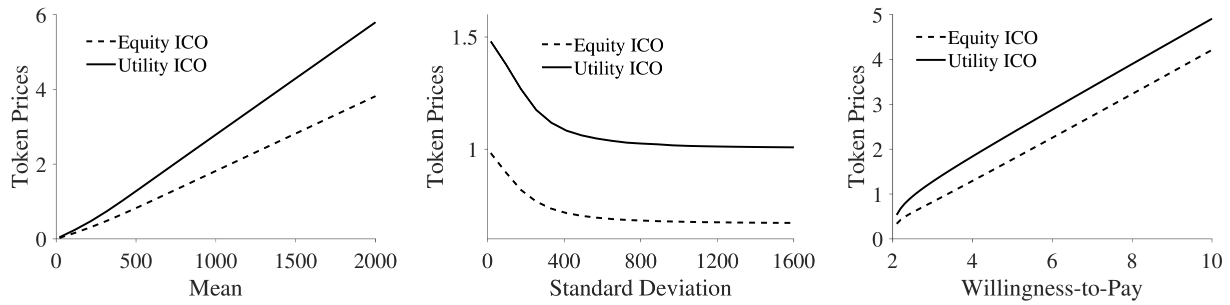
Another immediate implication of a higher production level with the issuance of equity tokens, by Proposition 3 (iii), is that the funds raised through the equity token ICO must surpass that through the utility token ICO, because $s(n_e^*, \tau_e^*) \cdot \tau_e^* \geq c \cdot Q_e^* > c \cdot Q^* = s(n^*, \tau^*) \cdot \tau^*$.

Figure 6 shows that the revenue-sharing utility tokens have a higher market value than the equity tokens as the prices of the equity tokens (both the ICO token price and the expected market equilibrium token price) are consistently lower. Figure 7 shows that more equity tokens will be sold than utility tokens, although the gap diminishes under better market conditions. Since the



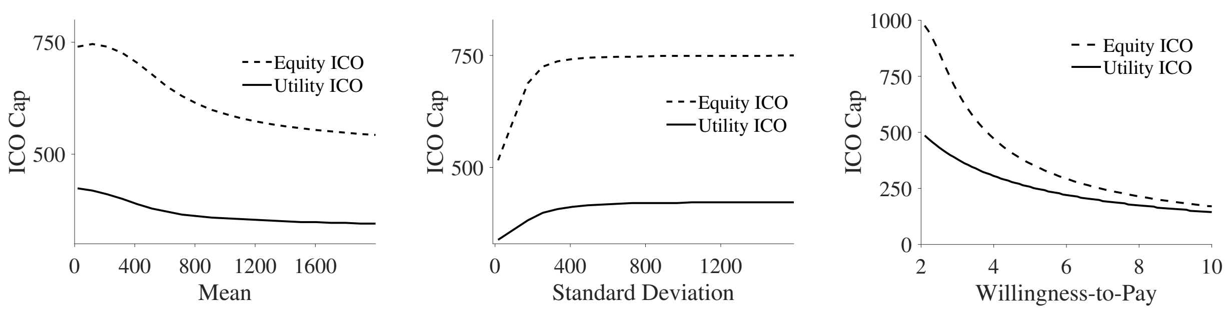
(a) Production Quantity vs Mean (b) Production Quantity vs Std (c) Production Quantity vs WTP

Figure 5: Comparison of Production Quantities



(a) Token Prices vs Mean (b) Token Prices vs Std (c) Token Prices vs WTP

Figure 6: Comparison of Token Prices

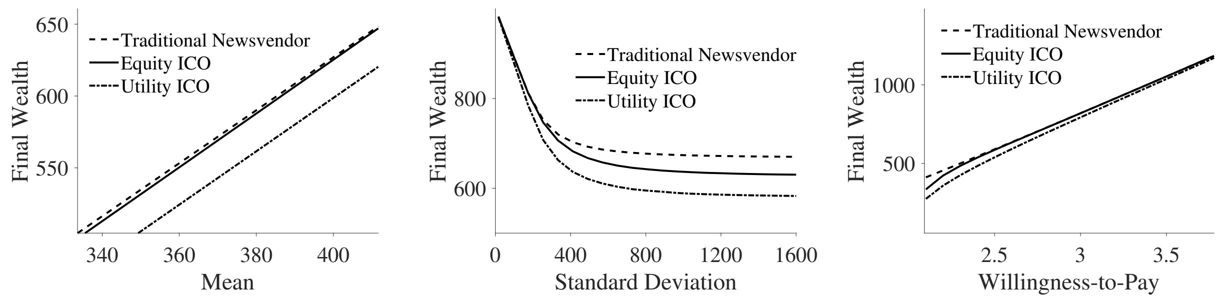


(a) ICO Cap vs Mean (b) ICO Cap vs Std (c) ICO Cap vs WTP

Figure 7: Comparison of ICO Caps

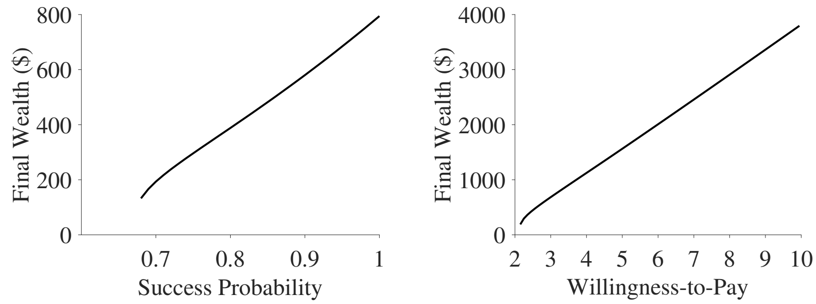
total ICO proceeds, which equals the product of price and ICO cap, is higher with equity tokens, the effect of a larger ICO cap outweighs that of lower prices. Moreover, it can be readily checked that the firm spends all ICO proceeds on production rather than leaving any funds idle, i.e., $s(n_e^*, \tau_e^*) \cdot \tau_e^* = n_e^* \cdot \tau_e^* = c \cdot Q_e^*$. Note that this result with equity tokens echoes that with utility tokens (Proposition 3 (iii)).

Finally, with a closer-to-optimal production quantity, the firm obtains a higher total wealth with equity tokens than with utility tokens (Figure 8). In particular, when market conditions are



(a) Final Wealth vs Mean (b) Final Wealth vs Std (c) Final Wealth vs WTP

Figure 8: Comparison of Final Wealth



(a) Final Wealth vs α ($v = 3$) (b) Final Wealth vs v ($\alpha = 0.95$)

Figure 9: Firm's Final Wealth under Risks of Production Failure

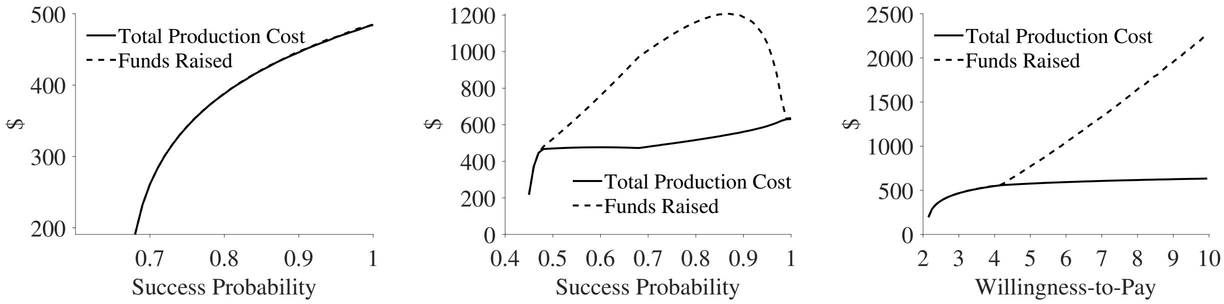
better, equity tokens allow the firm to achieve near-the-first-best outcome.

C.3 Numerical Experiments: Risk of Production Failure

Numerically, we show that the firm's final wealth increases in the success probability (Figure 9 (a)) and customers' willingness-to-pay (Figure 9 (b)). Interestingly, the optimal strategy varies for different values of α and v. Recall that we show in Proposition 3 (iii) that when there is no risk ($\alpha = 1$), the firm invests all money raised on production. Now, for $\alpha < 1$, the firm does the same if either the risks are high or the willingness-to-pay is low (Figure 10 (a,b)). Under more favorable conditions, i.e. low risk ($\alpha < 1$ but close to 1) and high willingness-to-pay, the firm spends part of its funds raised on production and saves the rest (Figure 10 (c)). Such practice guarantees that the firm ends up with positive final wealth even if production fails.

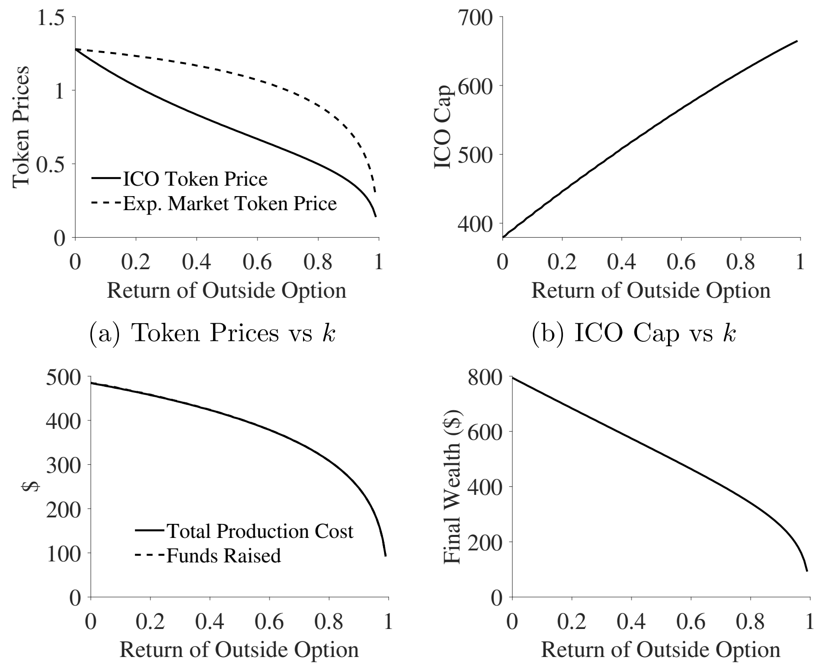
C.4 Numerical Experiments: Speculators with Outside Investment Options

Intuitively, a better-paying outside option makes ICOs less attractive in comparison. To incentivize the speculators' to participate, the firm needs to make token trading more lucrative by either raising



(a) Funds Usage vs α ($v = 3$) (b) Funds Usage vs α ($v = 6$) (c) Funds Usage vs v ($\alpha = 0.95$)

Figure 10: Funds Usage under Risks of Production Failure



(a) Token Prices vs k (b) ICO Cap vs k

(c) Funds Usage vs k (d) Firm's Final Wealth vs k

Figure 11: Impact of Outside Investment Return on ICOs

the expected market token price or reducing the ICO token price. Since the former is difficult to achieve given that the demand distribution remains unchanged, the firm has to do the latter. Our numerical results show that as k increases, the token prices drop (Figure 11 (a)) and the firm sells more tokens during the ICO (Figure 11 (b)) to mitigate the loss in funds raised. A higher k also discourages production (Figure 11 (c)) and hurts the firm's final wealth (Figure 11 (d)).

In Appendix B.6, we show that the optimal ICO token price makes the expected return of the tokens equal to that of the outside option. This result can be readily checked in Figure 11 (a), where for any value of k, the difference of the token prices divided by the ICO token price is exactly

k.

D Proofs

Proof of Lemma 1

i) First note that the customers have a fixed willingness-to-pay v that is equal to $p \cdot \tau_{eq}$. Suppose $p > m/\min\{Q, D\}$, then the demand of tokens $p \cdot \min\{Q, D\}$ exceeds the supply of tokens, m. This will drive the price of the token up, resulting in a decrease in the token-denominated price. In other words, τ_{eq} will increase and p will decrease. Similarly, if $p < m/\min\{Q, D\}$, then the demand of tokens is less than the supply of tokens, which induces an increase in p. Therefore, in equilibrium, demand of tokens is equal to its supply, i.e., $p \cdot \min\{Q, D\} = m$.

ii) The result follows immediately from $\tau_{eq} = v/p$ and Part (i).

□

Proof of Proposition 1

Taking derivative with respect to Q and applying Lemma 1,

$$\begin{aligned} \frac{d\Pi}{dQ} &= -c + (m-s)\frac{d}{dQ}\frac{v\mathbb{E}[\min\{Q, D\}]}{m} \\ &= -c + (m-s)\frac{v}{m}(1 - F(Q)) \\ &= [(m-s)\frac{v}{m} - c] - (m-s)\frac{v}{m}F(Q) \end{aligned} \tag{16}$$

By (16), $\frac{d\Pi}{dQ} < 0$ when $(m-s)\frac{v}{m} - c < 0$, i.e., $s > m\left(1 - \frac{c}{v}\right)$. On the other hand, when $s \leq m\left(1 - \frac{c}{v}\right)$, ignoring the budget constraint and setting $\frac{d\Pi}{dQ} = 0$, we get $Q^*_{unconstrained}(s) = F^{-1}(1 - \frac{cm}{(m-s)v})$. Since $\frac{d^2\Pi}{dQ^2} = -(m-s)\frac{v}{m}f(Q) < 0$, the profit function is concave in Q and $Q^*_{unconstrained}$ is a maximum. Hence the firm's optimal production quantity is given by

$$Q^*(s) = \min\left\{F^{-1}(1 - \frac{cm}{(m-s)v}), \frac{\tau\, s}{c}\right\} \cdot \mathbb{1}_{\{s \leq m(1-\frac{c}{v})\}}. \tag{17}$$

□

Proof of Proposition 2

i) For an ICO to succeed, there must be a positive number of speculators who invest. Therefore, the firm needs to set a (τ, n) pair that satisfies the speculators' participation constraint. Consider a fixed $n > 0$. A necessary condition for this n to induce a successful ICO is that there exists $\tau > 0$ such that $s^*(\tau, n) > 0$ and $u(s^*(\tau, n)) \geq 0$, which is a necessary condition for the existence of $s_0(\tau)$. Therefore, we will characterize such n while assuming the existence of $s_0(\tau)$.

Now, for the fixed $n > 0$, we divide the space of possible τ into two partitions, $T_1 = \{\tau \geq 0 : s_0(\tau) < n\}$ and $T_2 = \{\tau \geq 0 : s_0(\tau) \geq n\}$, and in each partition look for eligible $\tau > 0$, i.e., $s^*(\tau, n) > 0$ and $u(s^*(\tau, n)) \geq 0$.

(*Simultaneous,* T_1) When $n > s_0(\tau)$, with simultaneous arrival $s^* = 0$. Therefore, there is no eligible $\tau > 0$ in T_1.

(*Simultaneous,* T_2) Now we consider T_2 where $0 < n \leq s_0(\tau)$. First note that when $\tau = 0$, the firm raises no money and thus produces $Q^* = 0$. Therefore $u(s^*(0, n)) = 0$ and $0 \in T_2$. To find out if an eligible $\tau > 0$ exists in T_2, we need to know how $u(s^*(\tau, n))$ changes in $\tau \in T_2$. Under simultaneous arrivals, by (1) and (8) we have

$$\begin{aligned}
\left.\frac{du(s^*(\tau,n))}{d\tau}\right|_{\tau\in T_2} &= \frac{d}{d\tau}\left[\frac{n}{z}\Delta(n)\right] \\
&= \frac{n}{z}\left[\frac{v}{m}\frac{d}{d\tau}\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{cm}{(m-n)v}), \frac{\tau\, n}{c}\right\}] - 1\right] \\
&= \frac{n}{z}\left[\frac{v}{m}\frac{d}{d\tau}\mathbb{E}[\min\left\{D, \frac{\tau\, n}{c}\right\}]\cdot \mathbb{1}_{\{F^{-1}(1-\frac{cm}{(m-n)v})\geq \frac{\tau\, n}{c}\}}\right] \\
&\quad + \frac{n}{z}\left[\frac{v}{m}\frac{d}{d\tau}\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{cm}{(m-n)v})\right\}]\cdot \mathbb{1}_{\{F^{-1}(1-\frac{cm}{(m-n)v})< \frac{\tau\, n}{c}\}} - 1\right] \\
&= \frac{n}{z}\left[\frac{v}{m}(1-F(\frac{\tau\, n}{c}))\,\frac{n}{c}\cdot \mathbb{1}_{\{\tau\leq \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})\}}\right] \\
&\quad + \frac{n}{z}\left[\frac{v}{m}(1-F(F^{-1}(1-\frac{cm}{(m-n)v})))\cdot 0\cdot \mathbb{1}_{\{\tau> \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})\}} - 1\right] \\
&= \frac{n}{z}\left[\frac{v}{m}(1-F(\frac{\tau\, n}{c}))\,\frac{n}{c}\cdot \mathbb{1}_{\{\tau\leq \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})\}} - 1\right]. \qquad (18)
\end{aligned}$$

By the analysis of T_1 and (18), for $\tau > \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})$, the speculators' utility would either remain the same (if $\tau \in T_1$) or keep decreasing in τ (if $\tau \in T_2$) as $\frac{du(s^*(\tau,n))}{d\tau}|_{\tau\in T_2} = -\frac{n}{z} < 0$. For $\tau \leq \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})$, $u(s^*(\tau, n)$ is either zero (if $\tau \in T_1$) or keeps decreasing in τ (if $\tau \in T_2$) as $(1 - F(\frac{\tau\, n}{c})$ decreases in τ. Hence, to guarantee a positive number of speculators and thus non-negative profit, it is necessary and sufficient for the platform to set n such that $\frac{du(s^*(\tau,n))}{d\tau}|_{\tau=0} = \frac{n}{z}\left[\frac{v}{m}(1-F(\frac{0\cdot n}{c}))\,\frac{n}{c} - 1\right] > 0$, i.e., $n > \frac{m\,c}{v}$. In this case, $\exists\tau > 0$ s.t. $u(s^*(\tau, n)) > 0$. Note that by definition of $s_0(\tau)$, it must be that $s^*(\tau, n) < s_0(\tau)$ and thus $n < s_0(\tau)$, which means that this τ is indeed in T_2.

(*Sequential*) Consider the sequential arrivals assumption.

When $n > s_0(\tau)$, $s^*(\tau, n) = \min\{s_0(\tau), n\} = s_0(\tau)$ and $u(s^*(\tau, n)) = 0$. Ostensibly, there exists eligible τ's in T_1. However, we have assumed the existence of $s_0(\tau)$ and we need to make sure that it still holds. The existence of $s_0(\tau)$ depends on the behavior of $u(s^*(\tau, n))$ for $\tau \in T_2$. By (8) and (7), we have

$$\begin{aligned} \left.\frac{du(s^*(\tau,n))}{d\tau}\right|_{\tau\in T_2} &= \frac{d}{d\tau}\Delta(n) \\ &= \frac{v}{m}\,(1-F(\frac{\tau\, n}{c}))\,\frac{n}{c}\cdot \mathbb{1}_{\{\tau \le \frac{c}{n}F^{-1}(1-\frac{cm}{(m-n)v})\}} - 1. \end{aligned} \tag{19}$$

Note that (19) only differs from (18) by a scale of $\frac{n}{z}$. We then follow a similar argument as in part (*Simultaneous,* T_2) to show that $s_0(\tau)$ exists if and only if $n > \frac{m\,c}{v}$.

ii) By Part (i), $s^* \ge \frac{m\,c}{v}$. On the other hand, we showed in Section B.1 that $s^* < m\left(1-\frac{c}{v}\right)$. Therefore, the ICO fails if $m\left(1-\frac{c}{v}\right) \le \frac{m\,c}{v}$, i.e., $v \le 2c$.

□

Proof of Proposition 3

i) Shown by Proposition 2.

ii) (a) Shown by Proposition 6.

(b) By Lemma 2, $s^*(\tau^*, n^*) = n^* \cdot \mathbb{1}_{\{u(n^*)\ge 0\}}$. By definition of τ^* as in Proposition 5 part (i), we know that $u(n^*) = u(n^*, n^*, \tau^*) \ge 0$. The result follows.

(c) By Proposition 5, we know that there exists a unique $\hat{n} \in (\frac{m\,c}{v}, \frac{m}{2})$ such that the following holds:

- $\frac{v\,\hat{n}}{c\,m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-\hat{n})v})\right\}] = F^{-1}(1-\frac{c\,m}{(m-\hat{n})v})$;
- $F^{-1}(1-\frac{c\,m}{(m-\hat{n})v}) = \frac{\tau^*(\hat{n})\hat{n}}{c}$.

We show in the proof of Proposition 6 that this $\hat{n}$ is a global maximum point, which we call n^*. Hence, $\frac{v\,n^*}{c\,m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-n^*)v})\right\}] = \frac{\tau^*(n^*)n^*}{c}$, and the ICO token price is $\tau^* = \frac{v}{m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-n^*)v})\right\}]$.

(d) Following the proof of part (c) and substituting n^* and τ^* into Proposition 1 part (i), we have $Q^* = \min\left\{F^{-1}(1-\frac{c\,m}{(m-n^*)v}), \frac{\tau^* n^*}{c}\right\} = F^{-1}(1-\frac{c\,m}{(m-n^*)v})$.

(e) By definition of τ^* as in Proposition 5 part (i), we have $\mathbb{E}[\tau_{eq}] = \tau^*$. We obtain the result by part (c).

iii) By part (a), (c) and (e) of Proposition 3, we have $n^* \cdot \tau^* = Q^* \cdot c$.

□

Proof of Proposition 4

i) Shown by Proposition 3.

ii) By Proposition 3, $Q^*_{ICO} = F^{-1}(1 - \frac{c\,m}{(m-n^*)v})$. The optimal production quantity of a traditional newsvendor is $F^{-1}(1-\frac{c}{v})$. Since $\frac{m}{m-n^*} > 1$ and F^{-1} is an increasing function, we have $F^{-1}(1-\frac{c\,m}{(m-n^*)v}) < F^{-1}(1-\frac{c}{v})$.

iii) The ICO newsvendor's profit is given by $\Pi_{ICO} = \tau^*\, s^* - c\,Q^* + (m - s^*)\,\mathbb{E}[\tau_{eq}]$. By Proposition 3, $\tau^* = \mathbb{E}[\tau_{eq}]$, therefore

$$
\begin{aligned}
\Pi_{ICO} &= m\,\mathbb{E}[\tau_{eq}] - c\,Q^* \\
&= v\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-n^*)v})\right\}] - c\,F^{-1}(1-\frac{c\,m}{(m-n^*)v}) \\
&= \Pi_{traditional}(F^{-1}(1-\frac{c\,m}{(m-n^*)v})) \qquad (20)
\end{aligned}
$$

where $\Pi_{traditional}$ is the profit function of a traditional newsvendor defined as $\Pi_{traditional}(Q) = v\,\mathbb{E}[\min\{D, Q\}] - c\,Q$. We know that $\Pi_{traditional}(Q)$ is maximized by $F^{-1}(1-\frac{c}{v})$ which is greater than $F^{-1}(1-\frac{c\,m}{(m-n^*)v})$ by part (ii). Therefore $\Pi_{traditional}(F^{-1}(1-\frac{c\,m}{(m-n^*)v})) < \Pi_{traditional}(F^{-1}(1-\frac{c}{v}))$.

iv) The fact that the firm who finances through ICO does not invest its own money makes sure that it never suffers a loss. Indeed, following (20),

$$
\Pi_{ICO} = v\int_0^{F^{-1}(1-\frac{c\,m}{(m-n^*)v})} x f(x)dx \,+\, (\frac{c\,m}{m-n^*} - c)\,F^{-1}(1-\frac{c\,m}{(m-n^*)v}) > 0. \qquad (21)
$$

□

Proof of Lemma 2

i) See the main text.

ii) See the main text.

iii) Fix τ and n. Recall that by (1) that $u(s)$ and $\Delta(s)$ have the same sign. Therefore, we can also express $s_0(\tau)$ as $\max\{s \geq 0 : \Delta(s) = 0\}$. We now examine the behavior of $\Delta(s)$ as a function of s:

$$
\left.\frac{d\Delta(s)}{ds}\right|_{s<m\left(1-\frac{c}{v}\right)} = \frac{v}{m}\,[1 - F(Q^*(s))]\left.\frac{dQ^*(s)}{ds}\right|_{s<m\left(1-\frac{c}{v}\right)}, \qquad (22)
$$

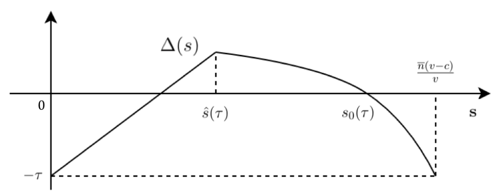
where

$$
\left.\frac{dQ^*(s)}{ds}\right|_{s\leq m\left(1-\frac{c}{v}\right)} = \begin{cases} -\frac{c\,m}{f(Q^*(s))(m-s)^2\,v} & \text{if } F^{-1}(1-\frac{cm}{(m-s)v}) \leq \frac{\tau\,s}{c} \\ \frac{\tau}{c} & \text{otherwise} \end{cases}. \qquad (23)
$$

Ignoring the sales cap n for the moment, note that for $s \in [0, m\left(1-\frac{c}{v}\right)]$, $F^{-1}(1-\frac{cm}{(m-s)v})$

monotonically decreases in s whereas $\frac{\tau s}{c}$ linearly increases in s. Also, $F^{-1}(1-\frac{cm}{(m-s)v})\big|_{s=0} = F^{-1}(1-\frac{c}{v}) > 0 = \frac{\tau s}{c}\big|_{s=0}$ and $F^{-1}(1-\frac{cm}{(m-s)v})\big|_{s=m\left(1-\frac{c}{v}\right)} = 0 < \frac{\tau s}{c}\big|_{s=m\left(1-\frac{c}{v}\right)}$. Therefore, for any fixed τ, there exists one and only one $\hat{s}(\tau)$ that satisfies $F^{-1}(1-\frac{cm}{(m-\hat{s}(\tau)v}) = \frac{\tau\,\hat{s}(\tau)}{c}$. By (23), $Q^*(s)$ increases in s for $s \in [0, \hat{s}(\tau))$ and decreases in s for $s \in (\hat{s}(\tau), m\left(1-\frac{c}{v}\right))$, and is thus maximized at $\hat{s}(\tau)$. Therefore, (22) is positive for all $s \in [0, \hat{s}(\tau))$ and negative for all $s \in (\hat{s}(\tau), m\left(1-\frac{c}{v}\right))$ and $\hat{s}(\tau)$ maximizes $\Delta(s)$. Now note that $\Delta(0) = 0 - \tau = -\tau$ and $\Delta(m\left(1-\frac{c}{v}\right)) = 0 - \tau = -\tau$. This shows that $s_0(\tau) \in [\hat{s}(\tau), m\left(1-\frac{c}{v}\right))$ if it exists. Figure 12 illustrates the relationships between the quantities mentioned above when demand is uniformly distributed.

Figure 12: $\Delta(s)$ vs s, assuming existence of $s_0(\tau)$



□

Proof of Proposition 5

i) First note that by Lemma 2 or (9), for each τ, it is redundant to consider $n > s_0(\tau)$. Therefore, for each n, we can restrict our attention to the set $T_r = \{\tau > 0 : s_0(\tau) \geq n\}$. When $n \leq s_0(\tau)$, we have $s^*(\tau, n) = n$. We will first find $\tau^*(n) \in \mathbb{R}^+$ that maximizes (5) evaluated at $s^*(\tau, n) = n$ and then show that this $\tau^*(n)$ is in T_r. Since $T_r \subset \mathbb{R}^+$, this $\tau^*(n)$ must maximize (5) over T_r.

Substituting $s^*(\tau, n) = n$ into (5) and differentiating with respect to τ,

$$\begin{aligned}
\frac{d\Pi}{d\tau} &= n - c\,\frac{dQ^*(n)}{d\tau} + (m-n)\,\frac{v}{m}\,\frac{d}{d\tau}\mathbb{E}[\min\{D, Q^*(n)\}] \\
&= n - c\,\frac{dQ^*(n)}{d\tau} + (m-n)\,\frac{v}{m}\,(1 - F(Q^*(n)))\,\frac{dQ^*(n)}{d\tau} \\
&= n + \left[(m-n)\,\frac{v}{m}\,(1 - F(Q^*(n))) - c\right]\frac{dQ^*(n)}{d\tau} \\
&= n + \left[(m-n)\,\frac{v}{m}\,(1 - F(Q^*(n))) - c\right]\mathbb{1}_{\{F^{-1}(1-\frac{cm}{(m-n)v}) \geq \frac{\tau n}{c}\}}\,\frac{n}{c} \\
&= \begin{cases} n + \left[(m-n)\,\frac{v}{m}\,(1 - F(Q^*(n))) - c\right]\frac{n}{c} & \text{if } F^{-1}(1-\frac{cm}{(m-n)v}) \geq \frac{\tau n}{c} \\ n & \text{otherwise} \end{cases}. \qquad (24)
\end{aligned}$$

Note that $F^{-1}(1-\frac{cm}{(m-n)v}) \geq \frac{\tau n}{c}$ means $(m-n)\,\frac{v}{m}\,(1 - F(Q^*(n))) - c \geq 0$. Therefore, $\frac{d\Pi}{d\tau} > 0$

for all τ, implying that for a given n, the optimal initial token price $\tau^*(n)$ is given by

$$\tau^*(n) = \max\{\tau : u(s^*(\tau, n)) = \mathbb{E}[\tau_{eq}(Q^*(n))] - \tau \geq 0\}. \tag{25}$$

Consider some $n \in (\frac{m\,c}{v}, m\left(1 - \frac{c}{v}\right)]$ and we know by (18) that $\frac{du(s^*(\tau,n))}{d\tau} > 0$ for $\tau \in [0, \tilde{\tau})$ for some $0 < \tilde{\tau} < \frac{c}{n}F^{-1}(1 - \frac{cm}{(m-n)v})$ such that $\frac{du(s^*(\tau,n))}{d\tau}\big|_{\tau=\tilde{\tau}} = 0$. Given that $u(s^*(0, n)) = 0$, by definition of τ^* given by (25), we must have $\tau^*(n) > \tilde{\tau} > 0$. We also know that $\tau^*(n) < \infty$ because by (18), the speculators' utility will eventually go negative as τ increases given that $\frac{du(s^*(\tau,n))}{d\tau} < 0$ when $\tau > \frac{c}{n}F^{-1}(1 - \frac{cm}{(m-n)v})$. Therefore, $\tau^*(n) = \max\{\tau : u(s^*(\tau, n)) = 0\}$. Since $u(s^*(\tau, n)) \geq 0$ for all $\tau \in [0, \tau^*(n)]$ and decreases linearly in τ for $\tau > \tau^*(n)$, the equation $u(s^*(\tau, n)) = 0$ has one and only one nonzero solution. We can thus simplify the definition by writing $\tau^*(n) = \{\tau > 0 : u(s^*(\tau, n)) = 0\}$.

Last, this new definition of $\tau^*(n)$ makes sure that $n \leq s_0(\tau^*(n))$ because $s_0(\tau^*(n))$ is the largest s that gives $u(s) = 0$ by definition. Therefore, $s^*(\tau, n) = n$ still holds. We can then solve $u(s^*(\tau, n)) = \frac{s^*(\tau,n)}{z}\Delta(s^*(\tau, n))$ or equivalently $\Delta(s^*(\tau, n)) = \frac{v}{m}\,\mathbb{E}[\min\left\{D, F^{-1}(1 - \frac{cm}{(m-n)v}), \frac{\tau\,n}{c}\right\}] - \tau = 0$.

ii) For a fixed $n \in (\frac{m\,c}{v}, m\left(1 - \frac{c}{v}\right)]$, we define
- $\tau_1(n) = \frac{v}{m}\,\mathbb{E}[\min\left\{D, F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)\right\}]$;
- $\tau_2(n) : \{\tau > 0 : \phi(\tau) = \frac{v}{m}\,\mathbb{E}[\min\left\{D, \frac{\tau\,n}{c}\right\}] - \tau = 0\}$.

By part (i) we know that $\tau^*(n)$ is either equal to $\tau_1(n)$ or given by $\tau_2(n)$.

We first show that $\tau_2(n)$ is finite and unique. Consider $\phi(\tau) = \frac{v}{m}\,\mathbb{E}[\min\left\{D, \frac{\tau\,n}{c}\right\}] - \tau$ and $\phi'(\tau) = \frac{v}{m}\,\frac{n}{c}\,(1 - F(\frac{\tau\,n}{c})) - 1$. Note that $\phi(0) = 0$ and $\phi'(0) > 0$ since $n > \frac{m\,c}{v}$. For large τ, $\phi'(\tau) < 0$ as $\phi''(\tau) = -\frac{v}{m}\,\frac{n^2}{c^2}\,f(\frac{\tau\,n}{c}) < 0$ for all $\tau \geq 0$. Therefore, there exists exactly one $0 < \tau < \infty$, which is $\tau_2(n)$, that gives $\phi(\tau) = 0$. Also note that $\phi'(\tau_2(n)) < 0$ and we will use this result in the proof of Proposition 6.

Next, let's find out the expression of $\tau^*(n)$ for $n \in (\frac{m\,c}{v}, m\left(1 - \frac{c}{v}\right)]$. Let $g(n) = \frac{\tau_1(n)n}{c} - F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)$ and note that $g(n) > 0$ means $\tau^*(n) = \tau_1(n)$. If $g(n) = 0$, then $F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right) = \frac{\tau_1(n)n}{c}$ and thus $\mathbb{E}[\min\left\{D, F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)\right\}] = \mathbb{E}[\min\left\{D, \frac{\tau_2(n)n}{c}\right\}]$, which by definition implies that $\tau_1(n) = \tau_2(n) = \tau^*(n)$. Also, $g(n) < 0$ means $\tau^*(n) \neq \tau_1(n)$ and thus $\tau^*(n) = \tau_2(n)$. We will first look at $n \in (\frac{m\,c}{v}, \frac{m}{2}]$ and then $n \in (\frac{m}{2}, m\left(1 - \frac{c}{v}\right)]$ ($v > 2c$ guarantees that $\frac{m\,c}{v} < \frac{m}{2} < m\left(1 - \frac{c}{v}\right)$).

Consider $n \in (\frac{mc}{v}, \frac{m}{2}]$. Note that $g(\frac{mc}{v}) = \mathbb{E}[\min\{D, F^{-1}(\frac{v-2c}{v})\}] - F^{-1}(\frac{v-2c}{v}) < 0$ and we now show that $g(\frac{m}{2}) > 0$. Let $r = \frac{v}{c}$ and we know that $r > 2$. Define $\tilde{g}(r) = g(\frac{m}{2}) = \frac{v}{2c}\,\mathbb{E}[\min\{D, F^{-1}\left(1-\frac{2c}{v}\right)\}] - F^{-1}\left(1-\frac{2c}{v}\right) = \frac{r}{2}\,\mathbb{E}[\min\{D, F^{-1}\left(1-\frac{2}{r}\right)\}] - F^{-1}\left(1-\frac{2}{r}\right)$. When $r = 2$, $\tilde{g}(2) = 0$. For $r \geq 2$, $\tilde{g}(r)$ increases in r as $\tilde{g}'(r) = \frac{1}{2}\,\mathbb{E}[\min\{D, F^{-1}\left(1-\frac{2}{r}\right)\}] + \frac{r}{2}\left[1 - FF^{-1}\left(1-\frac{2}{r}\right)\right]\frac{d}{dr}F^{-1}\left(1-\frac{2}{r}\right) - \frac{d}{dr}F^{-1}\left(1-\frac{2}{r}\right) = \frac{1}{2}\,\mathbb{E}[\min\{D, F^{-1}\left(1-\frac{2}{r}\right)\}] > 0$ for $r \geq 2$. Therefore, $g(\frac{m}{2}) = \tilde{g}(r) > 0$ for all $r > 2$.

Next note that

$$
\begin{aligned}
g'(n) &= \frac{\tau_1(n)}{c} + \frac{v\,n}{c\,m}(1 - FF^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right))\frac{d}{dn}F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)] \\
&\quad - \frac{d}{dn}F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right) \\
&= \frac{v}{cm}\,\mathbb{E}\left[\min\left\{D, F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right)\right\}\right] + \frac{2n-m}{m-n}\frac{d}{dn}F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right) \qquad (26)
\end{aligned}
$$

Since $\frac{d}{dn}F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right) < 0$, when $n \leq \frac{m}{2}$, $g'(n) > 0$. Therefore, there must exist a unique $\hat{n} \in (\frac{mc}{v}, \frac{m}{2})$ such that $g(\hat{n}) = 0$. This means that $\tau^*(n) = \tau_2(n)$ for $n \in (\frac{mc}{v}, \hat{n})$, $\tau^*(n) = \tau_1(n)$ for $n \in (\hat{n}, \frac{m}{2}]$, and $\tau^*(n) = \tau_1(n) = \tau_2(n)$ when $n = \hat{n}$.

For $n \in (\frac{m}{2}, m\left(1-\frac{c}{v}\right)]$, we have $g(m\left(1-\frac{c}{v}\right)) = \frac{v-c}{c}\mathbb{E}[\min\{D, F^{-1}(0)\}] - F^{-1}(0) = 0$ and $g'(m\left(1-\frac{c}{v}\right)) = 0 + \frac{2n-m}{m-n}\frac{d}{dn}F^{-1}\left(1 - \frac{c\,m}{(m-n)v}\right) < 0$ by (26). Since we have shown that $g(\frac{m}{2}) > 0$, there must be either zero or more than one $\hat{n} \in (\frac{m}{2}, m\left(1-\frac{c}{v}\right))$ such that $g(\hat{n}) = 0$. To rule out multiple zeros in the range of $n \in (\frac{m}{2}, m\left(1-\frac{c}{v}\right)]$, a sufficient condition is that $g''(n) < 0$ for $n \in (\frac{m}{2}, m\left(1-\frac{c}{v}\right)]$. We can find $g''(n)$ from (26) and, after some algebra, simplify it as

$$
g''(n) = -\frac{c\,m}{f(F^{-1}(y))(m-n)^4 v}\left[3n + \frac{(2n-m)cm}{(m-n)v}\cdot\frac{f'(F^{-1}(y)}{(f(F^{-1}(y))^2}\right], \qquad (27)
$$

where $y = 1 - \frac{c\,m}{(m-n)v}$.

Last, we find a sufficient condition for $g''(n) < 0$ for $n \in (\frac{m}{2}, m\left(1-\frac{c}{v}\right)]$ or equivalently $y \in [0, 1-\frac{2c}{v})$. By (27), to make $g''(n) < 0$, it suffices to have $3n + \frac{(2n-m)cm}{(m-n)v}\cdot\frac{f'(F^{-1}(y)}{(f(F^{-1}(y))^2} > 0$, or

$$
\frac{f'(F^{-1}(y)}{(f(F^{-1}(y))^2} > -\frac{3n(m-n)v}{(2n-m)cm}. \qquad (28)
$$

Let $\frac{n}{m} = k$. Then $k = 1 - \frac{c}{(1-y)v}$ and we look at $k \in (\frac{1}{2}, 1-\frac{c}{v}]$. Then, the right hand side of (28) is equal to $-\frac{3v}{c}\cdot k\cdot\frac{1-k}{2k-1}$. Therefore, under our assumption, (28) holds.

□

Proof of Proposition 6

By Proposition 5, we know that there exists a unique $\hat{n} \in (\frac{m\,c}{v}, \frac{m}{2})$ such that the following holds:

- $\frac{v\,\hat{n}}{c\,m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-\hat{n})v})\right\}] = F^{-1}(1-\frac{c\,m}{(m-\hat{n})v})$;
- $F^{-1}(1-\frac{c\,m}{(m-\hat{n})v}) = \frac{\tau^*(\hat{n})\hat{n}}{c}$.

We will first show that this $\hat{n}$ is a local maximum point. Differentiating the objective function (6) with respect to n, we have

$$\frac{d\Pi}{dn} = \frac{d\tau^*(n)}{dn}\cdot n + \tau^*(n) - c\,\frac{dQ^*(n)}{dn} + (m-n)\,\frac{d\mathbb{E}[\tau_{eq}(n)]}{dn} - \mathbb{E}[\tau_{eq}(n)]. \tag{29}$$

By part (i), we know that $\tau^*(n) = \mathbb{E}[\tau_{eq}(n)]$ and consequently simplify (29) as

$$\begin{aligned}\frac{d\Pi}{dn} &= n\,\frac{d\tau^*(n)}{dn} - c\,\frac{dQ^*(n)}{dn} + (m-n)\,\frac{d\mathbb{E}[\tau_{eq}(n)]}{dn}\\ &= n\,\frac{d\tau^*(n)}{dn} - c\,\frac{dQ^*(n)}{dn} + (m-n)\frac{v}{m}(1-F(Q^*(n)))\frac{dQ^*(n)}{dn}\\ &= n\,\frac{d\tau^*(n)}{dn} + [(m-n)\frac{v}{m}(1-F(Q^*(n))) - c]\frac{dQ^*(n)}{dn}.\end{aligned} \tag{30}$$

Let's now evaluate $\frac{d\Pi}{dn}$ at $n=\hat{n}$. We know that $\tau^*(\hat{n}) = \frac{v}{m}\,\mathbb{E}[\min\left\{D, F^{-1}(1-\frac{c\,m}{(m-\hat{n})v})\right\}]$ and $Q^*(\hat{n}) = Q^*(\tau^*(\hat{n}),\hat{n}) = \min\left\{F^{-1}(1-\frac{cm}{(m-\hat{n})v}), \frac{\tau^*(\hat{n})\,\hat{n}}{c}\right\} = F^{-1}(1-\frac{cm}{(m-\hat{n})v})$. Therefore, $(m-\hat{n})\frac{v}{m}(1-F(Q^*(\hat{n}))) - c$ vanishes. Hence,

$$\begin{aligned}\left.\frac{d\Pi}{dn}\right|_{n=\hat{n}} &= \hat{n}\left.\frac{d\tau^*(n)}{dn}\right|_{n=\hat{n}} + 0\\ &= \frac{v\,\hat{n}}{m}(1-F(Q^*(\hat{n})))\left.\frac{dQ^*(n)}{dn}\right|_{n=\hat{n}}\\ &= \frac{c\,\hat{n}}{m-\hat{n}}\left.\frac{dQ^*(n)}{dn}\right|_{n=\hat{n}}\end{aligned} \tag{31}$$

$Q^*(n)$ is not differentiable at $n=\hat{n}$ and thus $\frac{dQ^*(n)}{dn}\big|_{n=\hat{n}}$ does not exist. However, we've shown in the proof of Lemma 2 part iii) that given $\tau^*(\hat{n})$, $\frac{dQ^*(\tau^*(\hat{n}),n)}{dn}\big|_{n<\hat{n}} > 0$ and $\frac{dQ^*(\tau^*(\hat{n}),n)}{dn}\big|_{n>\hat{n}} < 0$. Therefore we know that $\lim_{n\to\hat{n}^-}\frac{d\Pi}{dn} > 0$ and $\lim_{n\to\hat{n}^+}\frac{d\Pi}{dn} < 0$, suggesting that $\hat{n}$ maximizes profit locally.

Last, we will show that $\hat{n}$ is the global maximum point by showing that (30) is negative for $n \in (\hat{n}, m\left(1-\frac{c}{v}\right)]$ and positive for $[\frac{m\,c}{v}, \hat{n})$.

For $n \in (\hat{n}, m\left(1-\frac{c}{v}\right)]$, we have $F^{-1}(1-\frac{cm}{(m-n)v}) < \frac{\tau^*(n)\,n}{c}$, $Q^*(n) = F^{-1}(1-\frac{cm}{(m-n)v})$ so $(m-n)\frac{v}{m}(1-F(Q^*(n))) - c = 0$. Since $\frac{d\tau^*(n)}{dn} = \frac{v}{m}\,\frac{d}{dn}\mathbb{E}[\min\left\{D, F^{-1}\left(1-\frac{c\,m}{(m-n)v}\right)\right\}] < 0$, (30) is negative.

Now for $n \in (\frac{m\,c}{v}, \hat{n})$, we have $F^{-1}(1 - \frac{cm}{(m-n)v}) > \frac{\tau^*(n)\,n}{c}$ and $Q^*(n) = \frac{\tau^*(n)\,n}{c}$.

$$\begin{aligned}
\left.\frac{d\Pi}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} &= n\left.\frac{d\tau^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} + \left[(m-n)\frac{v}{m}(1-F(Q^*(n))) - c\right]\left.\frac{dQ^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} \\
&= \frac{n\,v}{m}\left[1-F(Q^*(\hat{n}))\right]\left.\frac{dQ^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} \\
&\quad + \left[(m-n)\frac{v}{m}(1-F(Q^*(n))) - c\right]\left.\frac{dQ^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} \\
&= \left[(m-n+n)\frac{v}{m}(1-F(Q^*(n))) - c\right]\left.\frac{dQ^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} \\
&= \left[v\left[1-F(Q^*(n))\right] - c\right]\left.\frac{dQ^*(n)}{dn}\right|_{\frac{m\,c}{v}<n<\hat{n}} \qquad (32)
\end{aligned}$$

Note that

$$\begin{aligned}
v\left[1-F(Q^*(n))\right] - c &= v\left[1-F(\frac{\tau^*(n)\,n}{c})\right] - c \\
&> v\left[(1-F(F^{-1}(1-\frac{cm}{(m-n)v}))\right] - c \\
&= \frac{cm}{m-n} - c \\
&> 0. \qquad (33)
\end{aligned}$$

and $\frac{dQ^*(n)}{dn}\Big|_{\frac{m\,c}{v}<n<\hat{n}} = \frac{\tau^*(n)}{c} + \frac{n}{c}\frac{d\tau^*(n)}{dn}\Big|_{\frac{m\,c}{v}<n<\hat{n}}$. Therefore, to show that (32) is positive, it suffices to show $\frac{d\tau^*(n)}{dn}\Big|_{\frac{m\,c}{v}<n<\hat{n}} > 0$. By Proposition 5, when $\frac{m\,c}{v} < n < \hat{n}$, $\frac{d\tau^*(n)}{dn} = \frac{v}{m}(1 - F(\frac{\tau^*(n)\,n}{c}))\left[\frac{\tau^*(n)}{c} + \frac{n}{c}\frac{d\tau^*(n)}{dn}\right]$. Rearranging, we have

$$\frac{d\tau^*(n)}{dn} = -\frac{\frac{v}{m}(1-F(\frac{\tau^*(n)\,n}{c}))\frac{\tau^*(n)}{c}}{\frac{v}{m}(1-F(\frac{\tau^*(n)\,n}{c}))\frac{n}{c} - 1} \qquad (34)$$

The denominator of (34) is equal to $\phi'(\tau^*(n))$ where ϕ is defined in the proof of Proposition 5 and we've shown that $\phi'(\tau^*(n)) < 0$. Therefore, (34) is positive and this completes the proof. □

Proof of Proposition 7

The case where $s_0(\tau)$ does not exist is trivial. Suppose that $s_0(\tau)$ exists. When $n > s_0(\tau)$, by Lemma 2 part iii), we know that $s^*(\tau, n) = s_0(\tau)$ under sequential arrival. Now consider the case $n \le s_0(\tau)$. We show in the proof of Lemma 2 part iii) that $u(0) < 0$ and $u(s)$ is continuous and crosses zero at most once for $s \in [0, s_0(\tau))$. Therefore, if $u(n) < 0$, then $u(s) < 0$ for all $s \in [0, n]$. This means that no $s \leq \min\{s_0(\tau), n\}$ satisfies the participation constraint and hence $s^* = 0$. On the other hand, if $u(n) \geq 0$, then $s = n$ satisfies the participation constraint.

To see why (10) is a sufficient condition for s^* speculators, let's first consider the $s^* - th$ speculator that arrives after $s^* - 1$ other speculators have bought a token each. She knows that if she buys a token, then she will be the last person to do so — either because there is no extra token for sale ($s^* = n$) or buying tokens after her is no longer attractive ($s^* = m\left(1 - \frac{c}{v}\right)$). Therefore, (10) guarantees non-negative utility for her. Next, the $(s^* - 1) - th$ speculator knows that even if $u(s^* - 1) < 0$, buying a token now would induce the $s^* - th$ speculator to buy a token later, eventually resulting in non-negative rewards. By induction, we see that it is always optimal to buy a token for prior speculators.

□

Proof of Corollary 1

Substituting the expression of τ^* in Proposition 3 part c) into part a), we see that n^* and $\tau^*(n^*)$ satisfy $\frac{n^*}{c}\tau^*(n^*) = F^{-1}(1 - \frac{c\,m}{(m-n^*)v})$. Therefore, given $\tau^*(n^*)$, we know that $n^* = \hat{s}(\tau^*(n^*))$ where $\hat{s}(\tau)$ is the unique maximum point of $u(\tau, s)$ as defined in the proof of Lemma 2 part iii). Additionally, since $u(\tau^*(n^*), n^*) = 0$, we know that n^* is the only value of s such that $u(\tau^*(n^*), s) = 0$. Therefore, by definition of s_0, the result follows.

□

Proof of Proposition 8

Let Π_e denote the expected final wealth of the firm that issues equity tokens. Ignoring the budget constraint for the moment and taking derivative of Π_e with respect to Q, by (12),

$$\begin{aligned}
\frac{d\Pi_e}{dQ} &= v\,[1 - F(Q)] - c - \frac{s}{m}\frac{d}{dQ}\mathbb{E}[v\min\{Q, D\} - c\,Q]^+ \\
&= v\,[1 - F(Q)] - c - \frac{s}{m}\frac{d}{dQ}\left[(v - c)\,Q\,[1 - F(Q)] + \int_{\frac{c}{v}Q}^{Q}(v\,x - c\,Q)f(x)dx\right] \\
&= v\,[1 - F(Q)] - c - \frac{s}{m}\left[v\,[1 - F(Q)] - c + c\,F\left(\frac{c}{v}Q\right)\right] \\
&= \frac{m - s}{m}\,[v\,[1 - F(Q)] - c] - \frac{s\,c}{m}F\left(\frac{c}{v}Q\right). \qquad (35)
\end{aligned}$$

By (35), for $s \in (0, m)$, $\frac{d\Pi_e}{dQ}\big|_{Q=0} = \frac{m-s}{m}(v-c) - 0 > 0$ and $\frac{d^2\Pi_e}{dQ^2}\big|_{Q>0} = \frac{m-s}{m}[-f(Q)v] - \frac{s\,c}{m}\cdot\frac{c}{v}\,f\left(\frac{c}{v}Q\right) < 0$. Therefore, there exists a unique unconstrained optimal production quantity, denoted by $Q_u^*(s)$, such that $\frac{d\Pi_e}{dQ}\big|_{Q=Q_u^*(s)}=0$, i.e.,

$$\frac{m-s}{m}\,\left[v\left[1-F(Q_u^*(s))\right]-c\right] \,=\, \frac{s\,c}{m}\,F\left(\frac{c}{v}Q_u^*(s)\right). \tag{36}$$

Next, we show that $\frac{dQ_u^*(s)}{ds}<0$. Differentiating (36) with respect to s, we get

$$-(v-c)+v\,F(Q_u^*(s))-c\,F\left(\frac{c}{v}Q_u^*(s)\right) \,=\, \left[(m-s)v\,f(Q_u^*(s))+s\,c\,f\left(\frac{c}{v}Q_u^*(s)\right)\cdot\frac{c}{v}\right]\frac{dQ_u^*(s)}{ds}. \tag{37}$$

By (36), the left-hand side of (37) equals $-\frac{m}{s}[v\,(1-F(Q_u^*(s)))-c$, which is negative. Since the coefficient of $\frac{dQ_u^*(s)}{ds}$ on the right-hand side of (37) is positive, $\frac{dQ_u^*(s)}{ds}$ must be negative.

□

Proof of Proposition 9

i) To make the ICO successful, the firm needs to set a (τ_e,n_e) pair such that a positive number of speculators participate in the ICO, i.e., $s(\tau_e,n_e)>0$, which requires the participation constraint.

We first evaluate the behavior of $\Delta(s(\tau_e,n_e))$. Now, $\Delta(s(\tau_e,n_e))=\frac{1}{m}\mathbb{E}[v\min\{Q_e^*(s(\tau_e,n_e)),D\}-c\,Q_e^*(s(\tau_e,n_e))]^+-\tau_e$. For a fixed τ_e,

$$\begin{aligned}\frac{d\Delta(s)}{ds} &= \frac{1}{m}\,\frac{\partial}{\partial Q_e^*(s)}\mathbb{E}[v\min\{Q_e^*(s),D\}-c\,Q_e^*(s)]^+\,\frac{dQ_e^*(s)}{ds}\\ &= \frac{1}{m}\,\left\{v\left[1-F(Q_e^*(s))\right]-c+c\,F\left(\frac{c}{v}Q_e^*(s)\right)\right\}\,\frac{dQ_e^*(s)}{ds}.\end{aligned} \tag{38}$$

Following similar arguments as in Lemma 2 (iii) and the regularity assumption that $f(x)<a^2\cdot f(ax)$ for $a>2$, we can show that $v\left[1-F(Q_e^*(s))\right]-c+c\,F\left(\frac{c}{v}Q_e^*(s)\right)>0$ for all s. This, given that $\frac{dQ_u^*(s)}{ds}<0$, means that there exists a unique $\hat{s}(\tau_e)$ that satisfies $Q_u^*(s)=\frac{\tau_e\,\hat{s}(\tau_e)}{c}$ and $\hat{s}$ maximizes $\Delta(s)$.

Next, following the argument in Proposition 2 (i), we have

$$\begin{aligned}\left.\frac{du(s^*(\tau_e,n_e))}{d\tau_e}\right|_{\tau_e\in T_2} &= \frac{d}{d\tau_e}\left[\frac{n_e}{z}\Delta(n_e)\right]\\ &= \frac{n_e}{z}\left[\frac{1}{m}\,\frac{d}{d\tau_e}\mathbb{E}[v\min\{Q_e^*(n_e),D\}-c\,Q_e^*(n_e)]^+-1\right]\\ &= \frac{n_e}{z}\left[\frac{1}{m}\,\frac{\partial}{\partial Q_e^*}\mathbb{E}[v\min\{Q_e^*(n_e),D\}-c\,Q_e^*(n_e)]^+\,\frac{dQ_e^*}{d\tau_e}-1\right]\\ &= \frac{n_e}{z}\left[\frac{1}{m}\,\left\{v\left[1-F(Q_e^*(n_e))\right]-c+c\,F\left(\frac{c}{v}Q_e^*(n_e)\right)\right\}\,\frac{dQ_e^*}{d\tau_e}-1\right]\end{aligned} \tag{39}$$

Again, the firm needs $\frac{du(s^*(\tau_e,n_e))}{d\tau_e}\Big|_{\tau_e=0} = \frac{n_e}{z}\left[\frac{1}{m}\,\{v-c+0\}\,\frac{n_e}{c}-1\right] > 0$, i.e., $n_e > \frac{c}{v-c}\,m$.

ii) Since we need $n_e < m$, by part (i), we must have $1 > \frac{c}{v-c}$, i.e., $v > 2c$.

□

Proof of Proposition 10

For a fixed n_e, $\frac{d\Pi_e}{d\tau_e} = \frac{\partial\Pi_e}{\partial\tau_e} + \frac{\partial\Pi_e}{\partial Q_e^*}\frac{dQ_e^*}{d\tau_e} = n_e + \frac{\partial\Pi_e}{\partial Q_e^*}\frac{dQ_e^*}{d\tau_e}$. Note that $\frac{\partial\Pi_e}{\partial Q_e^*} > 0$ because $Q_e^* \leq Q_u^*$, and $\frac{dQ_e^*}{d\tau_e} = \frac{n_e}{c}$ or 0. Therefore, we know that $\frac{d\Pi_e}{d\tau_e} > 0$. Given that τ_e^* must satisfy the participation constraint, we have $u(s^*(\tau_e^*(n_e))) = 0$. By (39), we know that such τ_e^* is finite. Lastly, since $u(s^*(\tau_e, n_e)$ is linear in τ_e, $\tau_e^*(n_e)$ must be unique.

□

Proof of Proposition 11

Differentiate (14) with respect to Q,

$$\frac{d\Pi}{dQ} \;=\; [(m-s)\,\frac{\alpha\,v}{m}\,-\,c\,]\,-\,\alpha\,(m-s)\,\frac{v}{m}F(Q) \tag{40}$$

By (40), $\frac{d\Pi}{dQ} < 0$ when $\alpha(m-s)\,\frac{v}{m} - c < 0$, i.e., $s > m(1-\frac{c}{\alpha\,v})$. On the other hand, when $s \leq m(1-\frac{c}{\alpha\,v})$, ignoring the budget constraint and setting $\frac{d\Pi}{dQ} = 0$, we get $Q^*_{unconstrained}(s) = F^{-1}(1-\frac{cm}{\alpha(m-s)v})$. Since $\frac{d^2\Pi}{dQ^2} = -\,\alpha\,(m-s)\,\frac{v}{m}f(Q) < 0$, the profit function is concave in Q and $Q^*_{unconstrained}$ is a maximum. Hence the firm's optimal production quantity is given by

$$Q^*(s) = \min\left\{F^{-1}(1-\frac{cm}{\alpha\,(m-s)v}), \frac{\tau\,s}{c}\right\}\cdot \mathbb{1}_{\{s\leq m(1-\frac{c}{\alpha\,v})\}}. \tag{41}$$

□

Proof of Proposition 12

i) We substitute the new definition of the market equilibrium token price, $\tau_{eq} = \alpha\cdot\frac{v}{m}\min\{Q, D\}$, into (1), and then follow similar arguments in the proofs of Lemma 2(iii) and Proposition 2. Applying (41), we have

$$\begin{aligned}\frac{du(s^*(\tau,n))}{d\tau}\bigg|_{\tau\in T_2} &= \frac{d}{d\tau}\left[\frac{n}{z}\Delta(n)\right]\\ &= \frac{n}{z}\left[\frac{\alpha\,v}{m}\,(1-F(\frac{\tau\,n}{c}))\,\frac{n}{c}\cdot\mathbb{1}_{\{\tau\leq\frac{c}{n}F^{-1}(1-\frac{cm}{\alpha\,(m-n)v})\}}-1\right].\end{aligned} \tag{42}$$

By the analysis of T_1 and (42), for $\tau > \frac{c}{n}F^{-1}(1-\frac{cm}{\alpha(m-n)v})$, the speculators' utility would either remain the same (if $\tau\in T_1$) or keep decreasing in τ (if $\tau\in T_2$) as $\frac{du(s^*(\tau,n))}{d\tau}|_{\tau\in T_2} = -\frac{n}{z} < 0$. For $\tau \leq \frac{c}{n}F^{-1}(1-\frac{cm}{\alpha(m-n)v})$, $u(s^*(\tau,n))$ is either zero (if $\tau\in T_1$) or keeps decreasing in τ (if

$\tau \in T_2$) as $(1 - F(\frac{\tau n}{c})$ decreases in τ. Hence, to guarantee a positive number of speculators and thus non-negative profit, it is necessary and sufficient for the platform to set n such that $\frac{du(s^*(\tau,n))}{d\tau}\big|_{\tau=0} = \frac{n}{z}\left[\frac{\alpha v}{m}(1 - F(\frac{0 \cdot n}{c}))\frac{n}{c} - 1\right] > 0$, i.e., $n > \frac{m\,c}{\alpha v}$. In this case, $\exists \tau > 0$ s.t. $u(s^*(\tau, n)) > 0$. Note that by definition of $s_0(\tau)$, it must be that $s^*(\tau, n) < s_0(\tau)$ and thus $n < s_0(\tau)$, which means that this τ is indeed in T_2.

ii) By Part (i), $s^* \geq \frac{m\,c}{\alpha v}$. On the other hand, we showed in Appendix B.5 that $s^* < m(1 - \frac{c}{\alpha\, v})$. Therefore, the ICO fails if $m(1 - \frac{c}{\alpha\, v}) \leq \frac{m\,c}{\alpha\, v}$, i.e., $v \leq \frac{2c}{\alpha}$.

□

Proof of Proposition 13

i) We substitute the new definition of the expected profit given by (15) into (1), and then follow similar arguments in the proofs of Lemma 2(iii) and Proposition 2.

Applying (17), we have

$$\begin{aligned}\frac{du(s^*(\tau,n))}{d\tau}\bigg|_{\tau \in T_2} &= \frac{d}{d\tau}\left[\frac{n}{z}\Delta(n)\right] \\ &= \frac{n}{z}\left[\frac{v}{m}(1 - F(\frac{\tau\, n}{c}))\frac{n}{c} \cdot \mathbb{1}_{\{\tau \leq \frac{c}{n}F^{-1}(1 - \frac{cm}{(m-n)v})\}} - (1+k)\right]. \qquad (43)\end{aligned}$$

By the analysis of T_1 and (43), for $\tau > \frac{c}{n}F^{-1}(1 - \frac{cm}{(m-n)v})$, the speculators' utility would either remain the same (if $\tau \in T_1$) or keep decreasing in τ (if $\tau \in T_2$) as $\frac{du(s^*(\tau,n))}{d\tau}|_{\tau \in T_2} = -\frac{n}{z}(1 + k) < 0$. For $\tau \leq \frac{c}{n}F^{-1}(1 - \frac{cm}{(m-n)v})$, $u(s^*(\tau, n))$ is either zero (if $\tau \in T_1$) or keeps decreasing in τ (if $\tau \in T_2$) as $(1 - F(\frac{\tau n}{c})$ decreases in τ. Hence, to guarantee a positive number of speculators and thus non-negative profit, it is necessary and sufficient for the platform to set n such that $\frac{du(s^*(\tau,n))}{d\tau}\big|_{\tau=0} = \frac{n}{z}\left[\frac{v}{m}(1 - F(\frac{0 \cdot n}{c}))\frac{n}{c} - (1 + k)\right] > 0$, i.e., $n > \frac{m\,c}{v}(1 + k)$. In this case, $\exists \tau > 0$ s.t. $u(s^*(\tau, n)) > 0$. Note that by definition of $s_0(\tau)$, it must be that $s^*(\tau, n) < s_0(\tau)$ and thus $n < s_0(\tau)$, which means that this τ is indeed in T_2.

ii) By Part (i), $s^* \geq \frac{m\,c}{v}(1 + k)$. On the other hand, we showed in Appendix B.6 that $s^* < m\left(1 - \frac{c}{v}\right)$. Therefore, the ICO fails if $m\left(1 - \frac{c}{v}\right) \leq \frac{m\,c}{v}(1 + k)$, i.e., $v \leq (2 + k)c$.

□

Proof of Proposition 14

Since the firm's objective function remains unchanged by adding the outside option, the proof of this proposition resembles that of Proposition 5 (i). □