Engineering Systems for Allocating Public Goods

Homework

Due: Before Session 2

Concept Check

Question 1 (1 point) *Which of the following pieces of information are needed to determine whether an allocation is Pareto Efficient? (Check all that apply.)*

☐ *The set of agents and objects.*

☐ *The number of objects of each type.*

☐ *How agents rank objects.*

☐ *How strongly agents prefer their first choice.*

☐ *The order in which agents will choose.*

Question 2 (1 point) *Which of the following are correct restatements of the definition of Pareto efficiency? (Check all that apply.)*

☐ *It is impossible to (strictly) improve the outcome for anybody.*

☐ *It is impossible to (strictly) improve the outcome for everybody.*

☐ *It is impossible to (strictly) improve the outcome for anybody without (strictly) worsening the outcome for somebody else.*

☐ *It is impossible to improve the sum of ranks.*

Question 3 (4 points) *Suppose that we have one copy each of items A, B, C, D, and preferences are as follows:*

Agent 1: $A > B > C > D$.
Agent 2: $A > D > C > B$.
Agent 3: $B > C > D > A$.
Agent 4: $B > A > D > C$.

Which of the following allocations are Pareto optimal?

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(a) 1A, 2D, 3B, 4C

(b) 1B, 2A, 3C, 4D

(c) 1B, 2D, 3C, 4A

(d) 1C, 2D, 3B, 4A

Question 4 (4 points) *For each allocation considered in Question 3, can it be produced by serial dictatorship? If yes, what order of agents produces the allocation? If no, why not?*

(a) 1A, 2D, 3B, 4C is produced by order

(b) 1B, 2A, 3C, 4D is produced by order

(c) 1B, 2D, 3C, 4A is produced by order

(d) 1C, 2D, 3B, 4A is produced by order

Reflection and Critical Thinking

Question 5 (2 points) *There are (at least) two different ways to run serial dictatorship. The first is as we did in class: call people up one by one to select their most preferred remaining goption. This is called a **dynamic** implementation. The second is to ask everyone to submit a ranking of objects, and then to simulate the choosing process and announce the final allocation. This is called a **direct** implementation.*

Think of an example where a serial dictatorship has been used. This can be a system that you have participated in, read about, or heard about from friends or family. Was it run as a dynamic mechanism, or a direct mechanism? Would it be possible to change to the other format?

Question 6 (2 points) *Suppose that you are allocating 5 distinct prizes to 1000 applicants using a serial dictatorship. Would you prefer to use a dynamic implementation or a direct implementation? Why?*

Question 7 (3 points) *Can you come up with an example with four agents and four items in which there is only one Pareto efficient allocation? If so, describe it. If not, explain what you have tried.*

Question 8 (3 points) *Can you come up with an example with four agents and four items in which every allocation is Pareto efficient? If so, describe it. If not, explain what you have tried.*

Extra Credit

Question 9 *Can you construct an example of a Pareto efficient allocation in which no agent receives his or her first choice? If so, describe it. If not, explain what you have tried.*

Question 10 *When using serial dictatorship, every order of agents results in a Pareto efficient allocation. Do you think that every Pareto efficient allocation is discovered by some order of serial dictatorship, or are there some Pareto efficient allocations that serial dictatorship will never find? Explain your thinking.*