

# Engineering Systems for Allocation Public Goods

Homework Due Before Session 3

## Concept Check

**Question 1 (3 points)** Suppose that there are four agents (1, 2, 3, 4). Initially, agent 1 owns object C, agent 2 owns object B, agent 3 owns object A, and agent 4 owns object D. Preferences are as follows:

Agent 1 :  $A \succ B \succ C \succ D$ .

Agent 2 :  $A \succ D \succ C \succ B$ .

Agent 3 :  $B \succ C \succ D \succ A$ .

Agent 4 :  $B \succ A \succ D \succ C$ .

You are considering reallocating the objects so that 1 gets A, 2 gets C, 3 gets B, and 4 gets D. Which of the following are true? (Check all that apply.)

- The allocation ACBD is individually rational.
- The allocation ACBD is Pareto efficient.
- The allocation ACBD is in the core.

**Question 2 (1 point)** For the preceding example, come up with an allocation which is Pareto efficient but not individually rational.

**Question 3 (4 points)** Suppose that there are 8 agents and 8 objects, and preferences are as follows.

1	2	3	4	5	6	7	8
D	B	G	H	B	A	D	C
G	D	F	D	C	E	F	E
B	C	A	E	G	G	A	D
E	F	C	A	E	H	G	G
F	G	E	G	F	C	B	B
C	E	H	B	A	F	C	F
H	H	D	C	H	B	H	A
A	A	B	F	D	D	E	H

The initial allocation is given in red (FGECADHB), and you plan to apply Top Trading Cycles. What is the final allocation?

**Question 4 (2 points)** *You are in charge of running a math trade. What algorithm would you use, and why?*

*Note that no algorithm is perfect, so there are multiple “correct” answers. I am most interested in your justification. When writing your answer, include as many vocabulary terms from the study guides as possible (I want you to practice with these concepts).*