Engineering Systems for Allocating Public Goods

Homework Due Before Session 4

1 Concept Check

Question 1 (2 points) *Suppose that preferences are as given below, with the initial allocation AFCDHGBE shown in red. If we use Top Trading Cycles, what is the final allocation?*

1	2	3	4	5	6	7	8
D	B	G	H	B	A	D	C
G	D	F	D	C	E	F	E
B	C	A	E	G	G	A	D
E	F	C	A	E	H	G	G
F	G	E	G	F	C	B	B
C	E	H	B	A	F	C	F
H	H	D	C	H	B	H	A
A	A	B	F	D	D	E	H

Question 2 (2 points) *Suppose that you are given a mystery mechanism M. You know preferences are as follows:*

$$1 : A \succ B \succ C \succ D$$
$$2 : A \succ D \succ B \succ C$$
$$3 : B \succ C \succ A \succ D$$
$$4 : B \succ A \succ D \succ C.$$

On this profile, the mechanism M produces the following random allocation: DCBA with probability 0.5, CABD with probability 0.3, and BACD with probability 0.2. Is the mechanism M Pareto efficient?

☐ *Yes*

☐ *No*

☐ *There is not enough information to decide.*

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Question 3 (2 points) *Consider the mechanism M from the previous question. Suppose that in addition, you are promised that this mechanism is symmetric.*

When preferences are as given below, what is the probability that the mechanism produces allocation CBAD? (Hint: compare this preference profile to the original.)

$$1 : A \succ B \succ C \succ D$$
$$2 : B \succ C \succ A \succ D$$
$$3 : A \succ D \succ B \succ C$$
$$4 : B \succ A \succ D \succ C.$$

☐ *0.5*

☐ *0.3*

☐ *0.2*

☐ *0*

☐ *There is not enough information to know.*

Question 4 (2 points) *Consider the mechanism M from the previous questions. Suppose that you also know that this mechanism is strategy proof (truthful).*

When preferences are as given below, what is the probability that the mechanism produces allocation ABCD? (Hint: compare this preference profile to the original.)

$$1 : D \succ A \succ B \succ C$$
$$2 : A \succ D \succ B \succ C$$
$$3 : B \succ C \succ A \succ D$$
$$4 : B \succ A \succ D \succ C.$$

☐ *0.5*

☐ *0.3*

☐ *0.2*

☐ *0*

☐ *There is not enough information to know.*

Question 5 (2 points) *Name (at least) two things that you hope to remember from this first unit.*

Reflection and Critical Thinking

So far, we have considered settings where no agent has claim to any object, and settings where every agent claims some object. This homework will explore a setting where some agents have an initial claim to objects, and others do not.

Specifically, we will be talking about the allocation of graduate student housing on the Stanford campus. Each year, some new students arrive, and some returning students continue to need housing. When I attended, the mechanism to allocate housing was roughly as follows.

1. Each returning student is asked whether they wish to stay in their current apartment, or give up their current apartment and enter the housing draw. Students who ask to stay in their current apartment are granted their wish. The remaining students are entered into the housing draw, and their former apartments are made available (along with any apartments vacated by graduating students).

2. Students in the draw (that is, new students and returning students who decided to give up their apartment) are asked to rank available apartments. They are placed in increasing order of seniority (so new students come first, followed by second-year students, followed by third-year students...), with a lottery to break ties. Serial dictatorship is applied in this order.

Question 6 (1 point) *Note that new students are simply asked to rank apartments. Is this mechanism truthful for new students?*

☐ *Yes*

☐ *No*

☐ *There is not enough information to know.*

☐ *I am not sure.*

Question 7 (2 points) *Returning students are first asked whether they want to keep their current housing, and only asked for their preferences if they answer 'no.' How do you think returning students will behave in this mechanism? Does you answer depend on factors such as the number of students, number of apartments, or the desirability of different apartments? If so, explain.*

Question 8 (2 points) *Do you think that the Stanford mechanism will result in a Pareto efficient allocation? Why or why not?*

Question 9 (1 point) *Consider the following modification to the Stanford mechanism.*
In step 1, returning students are asked whether they would like to retain their claim to their current apartment, similar to before.

In step 2, instead of using serial dictatorship, we will use top trading cycles. Furthermore, ALL students will participate, including those who asked to retain their claim (answered 'Yes') in step 1. These students will be given their current apartment as an initial allocation. New students and students who gave up their claim (answered 'No' in step 1) are assigned to a random available apartment. Then we apply top trading cycles.

Do you think that this approach will result in a Pareto efficient allocation?

Question 10 (2 points) *How do you think that returning students will think about their decision in Step 1 of the new mechanism? Could a returning student end up with an apartment that is worse than their current apartment?*

Question 11 (2 points) *Suppose that you want to eliminate Step 1, and move to a direct mechanism where all returning students simply rank graduate housing options. Can you come up with a direct mechanism which is truthful and individually rational?*

Question 12 (Extra Credit) *Can you come up with a direct mechanism for this setting which is truthful, individually rational, and Pareto efficient?*

For Monday's class, you are assigned to read two papers about school choice reforms in Boston and New York (available on Canvas).