

Engineering Systems for Allocating Public Goods

Session 4: Introduction to School Choice

Plan for Today

1. Homework Discussion: Allocating Graduate Housing
2. Discussion of School Choice in New York and Boston
3. School Choice Algorithms and Key Concepts

Allocating Graduate Housing

Could we give returning students priority for some vacant apartments?

Can we do this in a way that is individually rational, Pareto efficient, truthful?

Do Students Really Rank Apartments?

Stanford has *thousands* of graduate apartments!

Students rank apartment **types**:

- 4 graduate communities.
- 3 apartment types (4 bedroom, 2 bedroom, Studio).

Conflation: treating distinct objects as identical.

Generalized Top Trading Cycles

Inputs: **Preferences** for students, **priorities** and **capacity** for apartment types.

Matching Procedure:

1. Each student “points” to favorite apartment type with availability.
2. Each apartment type “points” to highest priority unassigned student.
3. Students in cycles get the apartment type they’re pointing to.
4. Repeat steps 1-3.

Generalizes:

- Serial Dictatorship (all apartments use same priorities)
- Top Trading Cycles (each student has highest priority for one apartment)

Generalized Top Trading Cycles

Inputs: **Preferences** for students, **priorities** and **capacity** for schools.

Matching Procedure:

1. Each student “points” to favorite school with availability.
2. Each school “points” to highest priority unassigned student.
3. Students in cycles get the school they’re pointing to.
4. Repeat steps 1-3.

Generalizes:

- Serial Dictatorship (all schools use same priorities)
- Top Trading Cycles (each student has highest priority for one school seat)

School Choice Discussion

What algorithms were used in Boston and New York?

Boston (pre-2005): First Choices First

New York (pre-2004): Dynamic Admissions

Concerns:

- Drawn out over months.
- 30,000 students administratively assigned!
- Non-transparent selection process.

School Choice Discussion

What algorithms were used in Boston and New York?

Were these algorithms symmetric?

What factors determined student priority?

What else did you find interesting about these assignment systems?

Break

New Fairness Notion: No Justified Envy

Given an assignment (allocation),

- Student 1 **envies** student 2 if
 - 1 would prefer to attend the school assigned to 2.
- Student 1 envy of student 2 is **justified** if
 - 1 has higher priority than 2 at the school assigned to 2.

An assignment **respects priorities** if there is **no justified envy**.

First Choices First Analysis

Student Preferences

1	2	3
A	B	A
B	A	B
C	C	C

School Priorities

A	B	C
2	1	1
1	3	2
3	2	3

Group Work: For the First Choices First algorithm,

1. What is the assignment?
2. Is this assignment Pareto efficient?
3. Do any students have justified envy?
4. Do any students have an incentive to misreport their preferences?

First Choices First Analysis

Student Preferences

1	2	3
A	B	A
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1. What is the assignment?
2. Is this assignment Pareto efficient?
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First Choices First in Practice

- Used in Boston (often called the “Boston” algorithm).

Abandoned 2005

"for a better may chance of getting your 'first choice' school... consider choosing less popular schools"

- Used in many English school districts.

Banned 2008

- Used in Chicago.

Abandoned 2009

- Many districts don't specify, likely use FCF.

Generalized Top Trading Cycles Analysis

Student Preferences

1	2	3
A	B	A
B	A	B
C	C	C

School Priorities

A	B	C
2	1	1
1	3	2
3	2	3

Group Work: For the Generalized Top Trading Cycle algorithm,

1. What is the assignment?
2. Is this assignment Pareto efficient?
3. Do any students have justified envy?
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Generalized Top Trading Cycles Analysis

Student Preferences

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Group Work: For the Generalized Top Trading Cycle algorithm,

1. What is the assignment?
2. Is this assignment Pareto efficient?
3. Do any students have justified envy?
4. Do any students have an incentive to misreport their preferences?

Generalized Top Trading Cycles in Practice

Used in New Orleans in 2012.

“To the best of our knowledge, [New Orleans] is the only place that TTC has been used in practice, in school assignment or elsewhere.”

Abandoned after one year!

1	2	3	A	B	C
A	B	A	2	1	1
B	A	B	1	3	2
C	C	C	3	2	3

Effect of Priorities in G-TTC:

- + At school with 100 seats, top 100 students guaranteed access (if they want it).
- Explain to Student 3 why Student 2 got School B.

Pareto Efficient + No Justified Envy

Student Preferences

1	2	3
A	B	A
B	A	B
C	C	C

School Priorities

A	B	C
2	1	1
1	3	2
3	2	3

Group Work: Find a Pareto efficient assignment which has no justified envy.

Non-wastefulness

An assignment is **non-wasteful** if no student prefers an under-enrolled school to their own assignment.



Important Questions

1. Is there always a non-wasteful assignment with no justified envy?
2. Could there be multiple assignments with these properties?
3. How do we find them?

Summary

School choice algorithms generally use **priorities** that differ by school.

First Choices First is a popular algorithm, but is not truthful. For this reason, it is no longer used in Boston, Chicago, and the UK.

Top Trading Cycles is Pareto efficient and truthful. It was tried in New Orleans, but quickly abandoned.

Neither of these algorithms respects priorities. Next class, we will discuss an algorithm that is non-wasteful and respects priorities.

Study Guide

Concepts

- Priority
- Conflation
- (No) Justified Envy
- Non-wasteful

Algorithms

- First Choices First (Boston)
- Generalized TTC

Facts

- Most districts limit applications, causing students to be assigned administratively, or in several rounds.
- No Justified Envy + Pareto efficient may be impossible.
- No Justified Envy + Non-wasteful always possible.
- Boston, Chicago, England stopped using FCF.
- New Orleans stopped using Generalized TTC.
- Many districts do not specify their algorithm.

Next Session

How to find non-wasteful assignments that have no justified envy?

Can we optimize over these assignments?

Can we do this in a way that is truthful?

Homework: Concept Check 4
(Posted this afternoon)