

# Engineering Systems for Allocating Public Goods

Homework Due Before Session 5

## Concept Check

The first four questions will address the following example. There are 8 students and 3 schools, each with 3 seats. Student preferences are:

- 1 :  $C \succ A \succ B$
- 2 :  $A \succ B \succ C$
- 3 :  $B \succ C \succ A$
- 4 :  $B \succ A \succ C$
- 5 :  $B \succ A \succ C$
- 6 :  $A \succ B \succ C$
- 7 :  $A \succ B \succ C$
- 8 :  $A \succ B \succ C$ .

School priorities are:

- $A$  :  $3 \succ 8 \succ 6 \succ 4 \succ 1 \succ 5 \succ 7 \succ 2$
- $B$  :  $7 \succ 3 \succ 5 \succ 2 \succ 4 \succ 1 \succ 8 \succ 6$
- $C$  :  $4 \succ 1 \succ 3 \succ 7 \succ 5 \succ 8 \succ 2 \succ 6$ .

**Question 1 (2 points)** *What is the assignment when using the First Choices First algorithm?*

**Question 2 (2 points)** *When using the First Choices First algorithm, find a student that could benefit from misreporting their preferences, and complete the following sentence: student \_\_\_\_\_ can obtain school \_\_\_\_\_ by submitting preference list \_\_\_\_\_.*

**Question 3 (2 points)** *What is the assignment when using the Generalized Top Trading Cycles algorithm?*

**Question 4 (2 points)** *Find a student who has justified envy when Generalized Top Trading Cycles is used, and complete the following sentence: student \_\_\_\_\_ has justified envy for student \_\_\_\_\_.*

**Question 5 (2 points)** Consider the  $3 \times 3$  example from class, in which schools have a single seat. Student preferences are:

$$1 : A \succ B \succ C$$

$$2 : B \succ A \succ C$$

$$3 : A \succ B \succ C$$

School priorities are:

$$A : 2 \succ 1 \succ 3$$

$$B : 1 \succ 3 \succ 2$$

$$C : 1 \succ 2 \succ 3.$$

There are six nonwasteful allocations. Which of them respect priorities? In other words, which have no justified envy?

$ABC$

$ACB$

$BAC$

$BCA$

$CAB$

$CBA$