Engineering Systems for Allocating Public Goods

Homework Due Before Session 6

Concept Check

Question 1 (1 point) *You are given an allocation, and must determine whether it is non-wasteful. What information might you need to know? (Check all that apply.)*

- ☐ *Capacities of each school.*
- ☐ *Which schools are listed by each student.*
- ☐ *How students rank schools.*
- ☐ *How students are ranked by schools.*

Question 2 (2 points) *There are three students and three schools, each with a single seat. Student preferences are as follows:*

$$
\begin{aligned}
1 :& A \succ B \\
2 :& B \succ A \\
3 :& B \succ C
\end{aligned}
$$

School priorities are

$$
\begin{aligned}
A :& 1 \succ 2 \succ 3 \\
B :& 2 \succ 1 \succ 3 \\
C :& 3 \succ 2 \succ 1
\end{aligned}
$$

Given these preferences and priorities, classify each of the following allocations. Note that 'X' indicates that the student gets nothing, so XXX represents the case where no student is assigned.

a) *ABC is (check one):*
☐ *Wasteful* ☐ *Non-Wasteful but not Pareto Efficient* ☐ *Pareto efficient.*

b) *BAC is (check one):*
☐ *Wasteful* ☐ *Non-Wasteful but not Pareto Efficient* ☐ *Pareto efficient.*

c) AXB is (check one):
☐ *Wasteful* ☐ *Non-Wasteful but not Pareto Efficient* ☐ *Pareto efficient.*

d) BXC is (check one):
☐ *Wasteful* ☐ *Non-Wasteful but not Pareto Efficient* ☐ *Pareto efficient.*

Question 3 (2 points) *There are 5 students and 2 schools, each with 2 seats. Student preferences are as follows:*

$$1 : A \succ B$$
$$2 : A \succ B$$
$$3 : B \succ A$$
$$4 : B \succ A$$
$$5 : B \succ A$$

School priorities are:

$$A : 3 \succ 2 \succ 1 \succ 4 \succ 5$$
$$B : 5 \succ 3 \succ 4 \succ 2 \succ 1$$
$$B : 5 \succ 3 \succ 4 \succ 2 \succ 1$$

You are considering assignment XAABB. That is, you are considering assigning students 2 and 3 to A, and students 4 and 5 to B. Find a pair that blocks this assignment, and complete the sentence below. Student _____ and school _____ form a blocking pair.

Question 4 (2 points) *There are 4 students and 4 schools, each with a single seat. Each student lists three schools, and student preferences are as follows:*

$$1 : A \succ B \succ C$$
$$2 : C \succ D \succ A$$
$$3 : B \succ C \succ A$$
$$4 : B \succ C \succ D$$

School priorities are:

$$A : 3 \succ 2 \succ 1 \succ 4$$
$$B : 1 \succ 4 \succ 3 \succ 2$$
$$C : 1 \succ 2 \succ 3 \succ 4$$
$$D : 2 \succ 4 \succ 1 \succ 3$$

What is the outcome of the student-proposing deferred acceptance algorithm?

Question 5 (2 points) *There are 4 students and 4 schools, each with a single seat. Each student took an entrance exam for three of the four schools. Student preferences are as follows:*

$$
\begin{aligned}
1 :& A \succ C \succ D \\
2 :& B \succ D \succ C \\
3 :& D \succ B \succ A \\
4 :& A \succ D \succ B
\end{aligned}
$$

Priorities are determined by school-specific exam scores, which are as follows:

$$
\begin{aligned}
A :& 3(82) \succ 4(75) \succ 1(71) \\
B :& 2(93) \succ 3(84) \succ 4(68) \\
C :& 2(87) \succ 1(72) \\
D :& 2(95) \succ 4(88) \succ 1(85) \succ 3(79)
\end{aligned}
$$

That is, student 3 scored 82 on A's entrance exam, and student 2 did not take A's entrance exam. If we use school proposing deferred acceptance, what are the final cutoffs at each school?

Question 6 (1 point) *In the previous question, what is the outcome of* **student***-proposing deferred acceptance?*

Reflection and Critical Thinking

Say that a mechanism is **bossy** *if it is possible for an agent to change their reported preferences in a way that does not change their own allocation, but does change others' allocation. A mechanism is* **non-bossy** *if this can never happen.*

Question 7 (1 point) *Is Serial Dictatorship bossy or non-bossy? If you say that it is bossy, give an example illustrating this point. If you say that it is non-bossy, explain your reasoning.*

Question 8 (2 points) *Is Student-Proposing Deferred Acceptance bossy or non-bossy? If you say that it is bossy, give an example illustrating this point. If you say that it is non-bossy, explain your reasoning.*

Question 9 (1 point) *In the stable roommates problem, there are an even number of potential roommates that must be paired. We will assume that each person ranks all others, so that all matchings are individually rational. A matching is* **stable** *if there is no pair of people that prefer each other to their match partner.*

Consider the following example. There are four people to be placed into two double rooms. Rankings are as follows:

$$1: 2 \succ 3 \succ 4$$
$$2: 3 \succ 4 \succ 1$$
$$3: 4 \succ 1 \succ 2$$
$$4: 1 \succ 2 \succ 3$$

Which matchings are stable? (Check all that apply.)

☐ *1 with 2, 3 with 4.*

☐ *1 with 3, 2 with 4.*

☐ *1 with 4, 2 with 3.*

Question 10 (2 points) *Consider other possible preference profiles for a 4-person stable roommates problem. Can you always find a stable matching? If yes, try to describe the algorithm you are using. If no, provide an example with no stable matching.*

Question 11 (2 points) *Consider a time when you applied for a position. This could be a job, an internship, or your college or graduate school applications. Was there a coordinated time during which offers were made or accepted?*

If you answered 'yes,' do you think participants could benefit by making offers before this time? What has prevented the market from unraveling?

If you answered 'no,' do you think that trying to coordinate offer timing would be a good idea? Why or why not?

Question 12 (2 points) *In class, we have talked about several desirable properties when allocating students to schools: Pareto efficiency, non-wastefulness, respecting priorities, and truthfulness. Unfortunately, it is not possible to guarantee all of these. For example, Pareto efficiency is sometimes in conflict with respecting priorities.*

Thinking about your own experiences, which of these properties do you think are most important, and why? Are there any that you do not think are important? If there are additional considerations that affect your answers (i.e. the number of students and schools, the structure of priorities, etc), feel free to mention them.