Engineering Systems for Allocating Public Goods

Plan for Today

1. Midterm, survey results, and homework review.
2. Why is there a student-optimal stable matching?
3. Is Deferred Acceptance truthful?
4. Dynamic implementations of Deferred Acceptance

First Midterm

24-hour take-home exam on Canvas.

Available Thursday February 17 through Sunday February 20.

Open notes.

Survey Results

Compared to your other classes, this class is

More work	Less work	Similar
5	1	14

Vast majority feel like concepts are new, but will feel comfortable with practice.

People like walking through algorithms.

Deferred Acceptance Algorithm

Always produces a stable assignment.

Used for residency matching since 1950s.

Key word: deferred.

Participants can always reconsider if better opportunities arise.

(Unlike first preferences first.)

Does Stability Matter?

Maybe similar benefits for any centralized clearinghouse?

STABLE AND UNSTABLE (CENTRALIZED) MECHANISMS

Market	Stable	Still in use (halted unraveling)
American medical markets		
NRMP	yes	yes (new design in '98)
Medical Specialties	yes	yes (about 30 markets)
British Regional Medical Markets		
Edinburgh ('69)	yes	yes
Cardiff	yes	yes
Birmingham	no	no
Edinburgh ('67)	no	no
Newcastle	no	no
Sheffield	no	no
Cambridge	no	yes
London Hospital	no	yes
Other healthcare markets		
Dental Residencies	yes	yes
Osteopaths (<'94)	no	no
Osteopaths (≥'94)	yes	yes
Pharmacists	yes	yes
Other markets and matching processes		
Canadian Lawyers	yes	yes (except in British Columbia since 1996)
Sororities	yes (at equilibrium)	yes

Evidence that stability leads to longevity (Roth 2002).

Homework Discussion: Non-Wastefulness

1: A > C

2: A > B

3: B > C

Is AXB non-wasteful?

Is AXB Pareto efficient?

Homework Discussion: Bossiness

A mechanism is **bossy** if it is possible for an agent to change their reported preferences in a way that does not change their own allocation, but does change others' allocation.

A mechanism is **non-bossy** if this can never happen.

Student-Proposing DA is bossy!

Every stable mechanism is bossy (Kojima 2010)

Student Answers	Bossy	Non-Bossy
Serial Dictatorship	0	20
Student-Proposing DA	6	14

School Priorities

A	B	C
2	1	1
1	3	2
3	2	3

3 reports truthfully

1	2	3
A	B	A
B	A	B
C	C	C

3 lists C first

1	2	3
A	B	C
B	A	A
C	C	B

Homework Discussion: Stable Roommates

We must pair four students. Roommate preferences are as follows:

1: 2 > 3 > 4

2: 3 > 1 > 4

3: 1 > 2 > 4

4: 1 > 2 > 3

Student Answers	Yes	No
Stable Match Always Exists?	14	6

Efficient algorithms can find stable matchings if they exist (Irving 1985).

Which pairings are stable?

None! Stable matchings may not exist in the roommate problem.

We should feel grateful that stable matchings exist in two-sided settings!

We should be grateful to world that math is nice!

Roommates: may be no stable match. Core does not exist! (Contrast to previous two cases.)

Couples: may be no stable match.

The “Marriage Problem”

Original terminology for 1-to-1 matching (men and women).

Amazing Facts

1. **Rural Hospital Theorem**: any two stable assignments assign the same students and the same number of seats at each hospital!

2. **Student Optimality:** all students agree that student-proposing DA gives the *best possible* stable match, and hospital-proposing DA gives the *worst possible* stable match!

Why is there a student-optimal stable matching?

Suppose that we have two stable matchings, yellow and blue.

We ask each student which matching they prefer (can be indifferent).

1 matches to A in both.

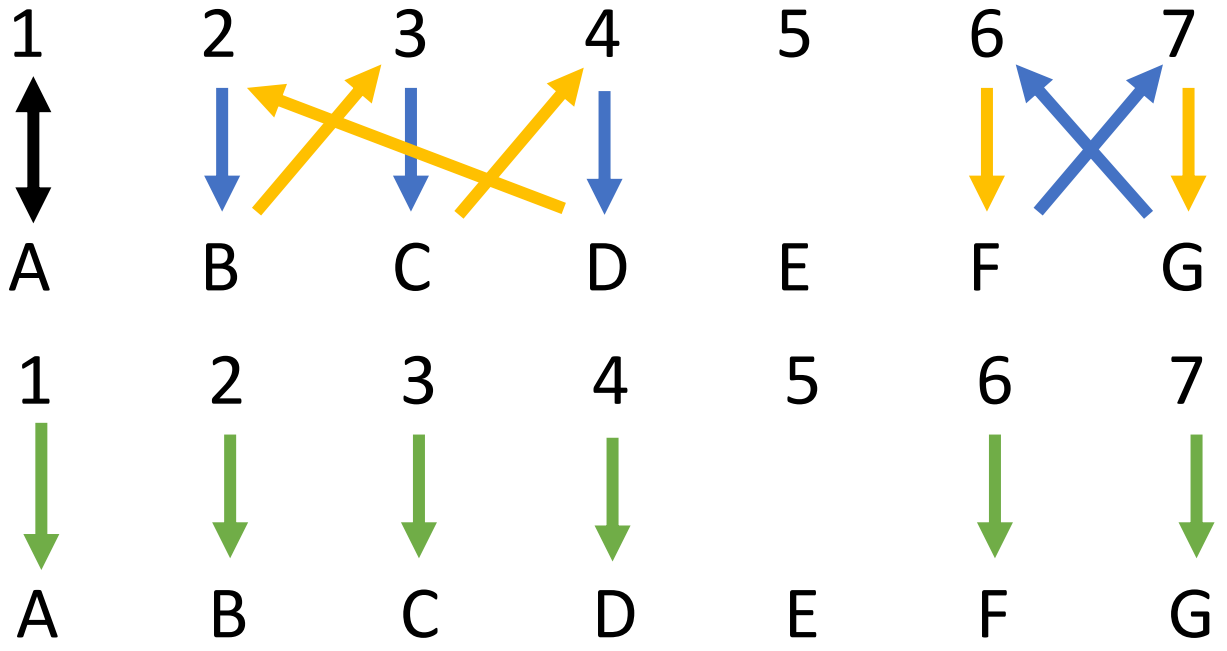
2 prefers blue, matches to B.

B must prefer yellow!

(Else yellow unstable)

3 must prefer blue!

(Else blue unstable)



In each cycle:

- Students agree which is better.
- Schools believe other is better.

Can create new stable matching which all students agree is better than yellow or blue.

Is Deferred Acceptance Truthful?

Student Preferences

1	2	3	4
B	C	C	C
D	B	B	D
A	D	D	B
C	A	A	A

School Priorities

A	B	C	D
3	3	1	1
1	4	3	2
4	1	2	4
2	2	4	3

Both algorithms:

- Result in DACB.
- No student benefits from lying.

Group Work:

1. What is the outcome of Student-Proposing Deferred Acceptance?
2. What is the outcome of School-Proposing Deferred Acceptance?
3. Under each mechanism, can any student benefit from misreporting?

Is Deferred Acceptance Truthful?

Student Preferences

1	2	3	4
A	B	D	A
C	D	B	D
D	C	A	B

School Priorities

A	B	C	D
3	2	2	2
4	3	1	4
1	4	3	1
2	1	4	3

Student-Proposing:

- Results in CBDA.
- No student benefits from lying.

School-Proposing:

- Results in CBAD.
- Student 3 (or 4) benefits from only listing first choice.

Group Work:

1. What is the outcome of Student-Proposing Deferred Acceptance?
2. What is the outcome of School-Proposing Deferred Acceptance?
3. Under each mechanism, can any student benefit from misreporting?

Cutoff Description of Stable Matching

Each school's cutoff = priority of lowest student to attend.

All students go to their favorite school where they clear the cutoff.

How could a student benefit from lying?

They could change the cutoffs!

1	2
A	B
B	A

A	B
2	1
1	2

School-Optimal Cutoffs

Student-Optimal Cutoffs

1	2
A	B
B	

A	B
2	1
1	2

Only Stable Cutoffs

School-Proposing DA is not truthful for students

A **truncation** strategy truthfully reports the top k of the list (for some k), but leaves remaining schools off the list.

Any student with multiple stable partners will get their least favorite under School-Proposing DA.

This student can guarantee their favorite stable partner by truncating their list below this option. (Eliminates worse stable matchings.)

Limited Benefit from Lying in School-Proposing DA

Students can never get better than their best stable partner by lying.

$\Rightarrow$ students with only one stable partner cannot benefit from lying!

Determining where to truncate is difficult.

If students truncate too far, may end up unassigned!

Student-Proposing DA is Truthful for Students

However, schools might benefit from lying.

No algorithm that always produces a stable assignment is truthful for both sides (Roth 1982).

Whichever side doesn't get their way will benefit from suitable truncation.

1	2
A	B
B	A

A	B
2	1
1	2

DA In Practice: List Limits

There may be no way to report your true preferences!

If using student-proposing DA with list limit:

- Students must strategize about which schools to list.
- Always best to list these schools in true preference order.

Same is **not** true of First Choices First.

Another version of Student-Proposing DA

Instead of proposing simultaneously, add one student at a time.

Outcome is same for any proposal order!

A	9	13	10	14	3	15	11	5	1	2	6	7	12	4	8
B	14	15	12	2	4	1	11	6	8	10	13	5	3	9	7
C	8	4	6	3	1	12	2	9	7	13	15	11	5	10	14

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
A	A	A	A	C	B	B	B	A	A	A	A	A	A	C
C	B	B	C	A	A	A	A	B	B	B	B	B	B	B
B	C	C	B	B	C	C	C	C	C	C	C	C	C	A

Population Monotonicity

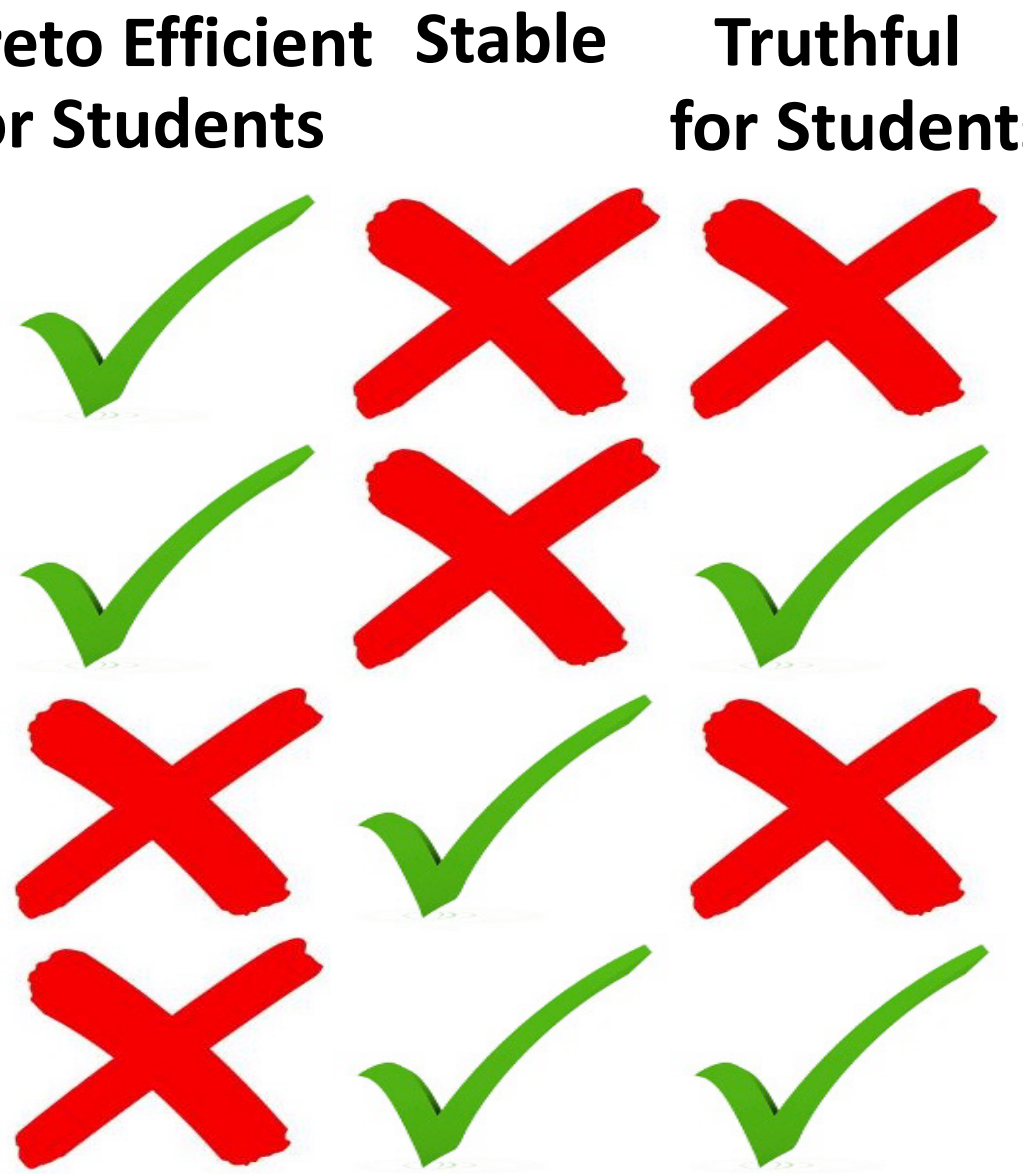
A mechanism is **population monotonic** if adding a new student to the market never improves outcomes for any other student.

Student-Proposing DA is population monotonic.

In fact, adding a school to the bottom of a student's list

- never results in a better outcome for any other student,
- never results in a worse outcome for any school.

Summary



No algorithm guarantees Stability and Pareto Efficiency (last week)
No algorithm guarantees Stability and Truthfulness for both sides.

Study Guide

Concepts

- Bossiness
- Population Monotonicity
- Truncation

Algorithms

- Student-Proposing DA (One at a time)

Facts

- Student-Proposing DA is population monotonic.
- Student-Proposing DA is truthful for students (not schools).
- No stable mechanism is truthful for both sides.
- No stable mechanism is non-bossy.

Next Class

Clinical Psychology Dynamic Match

- How much time is needed for market to clear?
- Do participants behave straightforwardly?

1990s Redesign of National Residency Matching Program

- Incorporating couples
- Which side proposes?